EENG282 HW01 – AUTHOR'S SOLUTIONS

NOTE: I have not yet verified that the author's solutions are, in fact, correct.

Prob. 9.8

P 9.8 $$V_{\rm rms} = \sqrt{\frac{1}{T}\int_0^{T/2} V_m^2 \sin^2 \frac{2\pi}{T} t\, dt}$$

$$\int_0^{T/2} V_m^2 \sin^2\left(\frac{2\pi}{T}t\right)\, dt = \frac{V_m^2}{2}\int_0^{T/2}\left(1 - \cos\frac{4\pi}{T}t\right)\, dt = \frac{V_m^2 T}{4}$$

$$\text{Therefore} \quad V_{\rm rms} = \sqrt{\frac{1}{T}\frac{V_m^2 T}{4}} = \frac{V_m}{2}$$

Prob. 9.17

P 9.17 [**a**] $Z_1 = R_1 + j\omega L_1$

$$Z_2 = \frac{R_2(j\omega L_2)}{R_2 + j\omega L_2} = \frac{\omega^2 L_2^2 R_2 + j\omega L_2 R_2^2}{R_2^2 + \omega^2 L_2^2}$$

$$Z_1 = Z_2 \quad \text{when} \quad R_1 = \frac{\omega^2 L_2^2 R_2}{R_2^2 + \omega^2 L_2^2} \quad \text{and} \quad L_1 = \frac{R_2^2 L_2}{R_2^2 + \omega^2 L_2^2}$$

[**b**] $$R_1 = \frac{(4000)^2(1.25)^2(5000)}{5000^2 + 4000^2(1.25)^2} = 2500\,\Omega$$

$$L_1 = \frac{(5000)^2(1.25)}{5000^2 + 4000^2(1.25)^2} = 625\,\text{mH}$$

Prob. 9.20

P 9.20 **[a]** $Y_2 = \frac{1}{R_2} + j\omega C_2$

$$Y_1 = \frac{1}{R_1 + (1/j\omega C_1)} = \frac{j\omega C_1}{1 + j\omega R_1 C_1} = \frac{\omega^2 R_1 C_1^2 + j\omega C_1}{1 + \omega^2 R_1^2 C_1^2}$$

Therefore $Y_1 = Y_2$ when

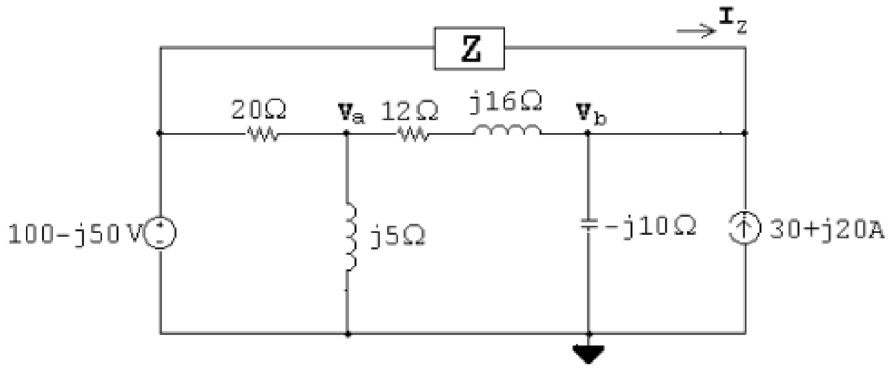
$$R_2 = \frac{1 + \omega^2 R_1^2 C_1^2}{\omega^2 R_1 C_1^2} \quad \text{and} \quad C_2 = \frac{C_1}{1 + \omega^2 R_1^2 C_1^2}$$

[b] $$R_2 = \frac{1 + (50 \times 10^3)^2 (1000)^2 (40 \times 10^{-9})^2}{(50 \times 10^3)^2 (1000) (40 \times 10^{-9})^2} = 1250\,\Omega$$

$$C_2 = \frac{40 \times 10^{-9}}{1 + (50 \times 10^3)^2 (1000)^2 (40 \times 10^{-9})^2} = 8\,\text{nF}$$

Prob. 9.35

P 9.35



$$\frac{\mathbf{V}_{\rm a} - (100 - j50)}{20} + \frac{\mathbf{V}_{\rm a}}{j5} + \frac{\mathbf{V}_{\rm a} - (140 + j30)}{12 + j16} = 0$$

Solving,

$$\mathbf{V}_{\rm a} = 40 + j30\,\mathrm{V}$$

$$\mathbf{I}_Z + (30 + j20) - \frac{140 + j30}{-j10} + \frac{(40 + j30) - (140 + j30)}{12 + j16} = 0$$

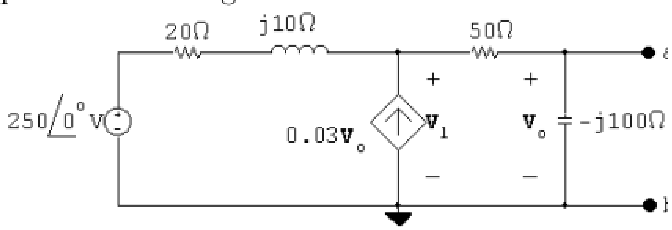
Solving,

$$\mathbf{I}_Z = -30 - j10\,\mathrm{A}$$

$$Z = \frac{(100 - j50) - (140 + j30)}{-30 - j10} = 2 + j2\,\Omega$$

Prob. 9.48

Open circuit voltage:



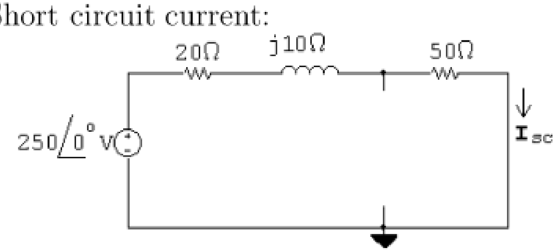
$$\frac{\mathbf{V}_1 - 250}{20 + j10} - 0.03\mathbf{V}_o + \frac{\mathbf{V}_1}{50 - j100} = 0$$

$$\therefore \quad \mathbf{V}_o = \frac{-j100}{50 - j100}\mathbf{V}_1$$

$$\frac{\mathbf{V}_1}{20 + j10} + \frac{j3\mathbf{V}_1}{50 - j100} + \frac{\mathbf{V}_1}{50 - j100} = \frac{250}{20 + j10}$$

$$\mathbf{V}_1 = 500 - j250\,\text{V}; \qquad \mathbf{V}_o = 300 - j400\,\text{V} = \mathbf{V}_{\text{Th}}$$

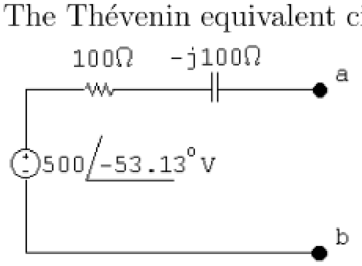
Short circuit current:



$$\mathbf{I}_{\text{sc}} = \frac{250\angle 0^\circ}{70 + j10} = 3.5 - j0.5\,\text{A}$$

$$Z_{\text{Th}} = \frac{\mathbf{V}_{\text{Th}}}{\mathbf{I}_{\text{sc}}} = \frac{300 - j400}{3.5 - j0.5} = 100 - j100\,\Omega$$

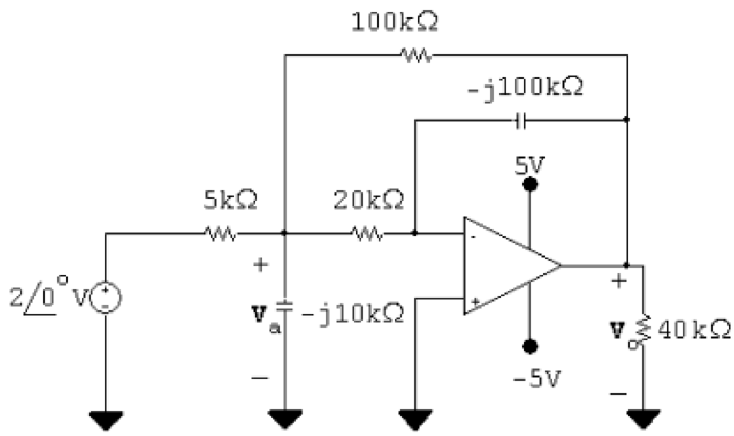
The Thévenin equivalent circuit:



Prob. 9.67

P 9.67 $\frac{1}{j\omega C_1} = -j10\,\text{k}\Omega$

$$\frac{1}{j\omega C_2} = -j100\,\text{k}\Omega$$



$$\frac{\mathbf{V}_\text{a} - 2}{5000} + \frac{\mathbf{V}_\text{a}}{-j10{,}000} + \frac{\mathbf{V}_\text{a}}{20{,}000} + \frac{\mathbf{V}_\text{a} - \mathbf{V}_o}{100{,}000} = 0$$

$$20\mathbf{V}_\text{a} - 40 + j10\mathbf{V}_\text{a} + 5\mathbf{V}_\text{a} + \mathbf{V}_\text{a} - \mathbf{V}_o = 0$$

$$\therefore \qquad (26 + j10)\mathbf{V}_\text{a} - \mathbf{V}_o = 40$$

$$\frac{0 - \mathbf{V}_\text{a}}{20{,}000} + \frac{0 - \mathbf{V}_o}{-j100{,}000} = 0$$

$$j5\mathbf{V}_\text{a} - \mathbf{V}_o = 0$$

Solving,

$$\mathbf{V}_o = 1.43 + j7.42 = 7.56\underline{/79.09^\circ}\ \text{V}$$

$$v_o(t) = 7.56\cos(10^6 t + 79.09^\circ)\ \text{V}$$

Prob. 9.79

P 9.79 $j\omega L_1 = j50\,\Omega$

$j\omega L_2 = j32\,\Omega$

$$\frac{1}{j\omega C} = -j20\,\Omega$$

$$j\omega M = j(4\times 10^3)k\sqrt{(12.5)(8)}\times 10^{-3} = j40k\,\Omega$$

$$Z_{22} = 5 + j32 - j20 = 5 + j12\,\Omega$$

$$Z_{22}^* = 5 - j12\,\Omega$$

$$Z_r = \left[\frac{40k}{|5+j12|}\right]^2 (5 - j12) = 47.337k^2 - j113.609k^2$$

$$Z_{\text{ab}} = 20 + j50 + 47.337k^2 - j113.609k^2 = (20 + 47.337k^2) + j(50 - 113.609k^2)$$

Z_{ab} is resistive when

$$50 - 113.609k^2 = 0 \quad \text{or} \quad k^2 = 0.44 \quad \text{so} \quad k = 0.66$$

$$\therefore \quad Z_{\text{ab}} = 20 + (47.337)(0.44) = 40.83\,\Omega$$