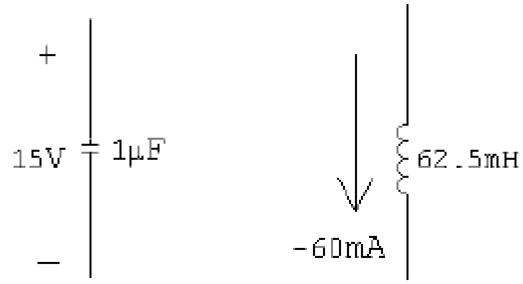
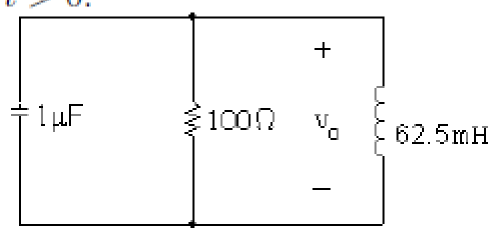
EENG 281 Homework #9 Solutions
Fall 2013

P 8.18 $t < 0: \qquad V_o = 15\,\text{V}, \qquad I_o = -60\,\text{mA}$



$t > 0$:



$$i_R(0) = \frac{15}{100} = 150\,\text{mA}; \qquad i_L(0) = -60\,\text{mA}$$

$$i_C(0) = -150 - (-60) = -90\,\text{mA}$$

$$\alpha = \frac{1}{2RC} = \frac{1}{2(100)(10^{-6})} = 5000 \text{ rad/s}$$

$$\omega_o^2 = \frac{1}{LC} = \frac{1}{(62.5 \times 10^{-3})(10^{-6})} = 16 \times 10^6$$

$$s_{1,2} = -5000 \pm \sqrt{25 \times 10^6 - 16 \times 10^6} = -5000 \pm 3000$$

$$s_1 = -2000 \text{ rad/s}; \qquad s_2 = -8000 \text{ rad/s}$$

$$\therefore \quad v_o = A_1 e^{-2000t} + A_2 e^{-8000t}$$

$$A_1 + A_2 = v_o(0) = 15$$

$$\frac{dv_o}{dt}(0) = -2000A_1 - 8000A_2 = \frac{-90 \times 10^{-3}}{10^{-6}} = -90{,}000$$

Solving, $\qquad A_1 = 5\,\text{V}, \qquad A_2 = 10\,\text{V}$

$$\therefore \quad v_o = 5e^{-2000t} + 10e^{-8000t}\,\text{V}, \qquad t \geq 0$$

P 8.29 $\omega_o^2 = \frac{1}{LC} = \frac{1}{(50\times10^{-3})(0.2\times10^{-6})} = 10^8; \qquad \omega_o = 10^4 \text{ rad/s}$

$$\alpha = \frac{1}{2RC} = \frac{1}{2(200)(0.2\times10^{-6})} = 12{,}500 \text{ rad/s} \qquad \therefore \quad \text{overdamped}$$

$$s_{1,2} = -12{,}500 \pm \sqrt{(12{,}500)^2 - 10^8} = -12{,}500 \pm 7500 \text{ rad/s}$$

$$s_1 = -5000 \text{ rad/s}; \qquad s_2 = -20{,}000 \text{ rad/s}$$

$$I_f = 60\,\text{mA}$$

$$i_\text{L} = 60\times10^{-3} + A_1'e^{-5000t} + A_2'e^{-20{,}000t}$$

$$\therefore \quad -45\times10^{-3} = 60\times10^{-3} + A_1' + A_2'; \qquad A_1' + A_2' = -105\times10^{-3}$$

$$\frac{di_\text{L}}{dt} = -5000A_1' - 20{,}000A_2' = \frac{15}{0.05} = 300$$

Solving, $\quad A_1' = -120\,\text{mA}; \qquad A_2' = 15\,\text{mA}$

$$i_\text{L} = 60 - 120e^{-5000t} + 15e^{-20{,}000t}\,\text{mA}, \quad t \geq 0$$

P 8.30 $\alpha = \dfrac{1}{2RC} = \dfrac{1}{2(312.5)(0.2 \times 10^{-6})} = 8000; \qquad \alpha^2 = 64 \times 10^6$

$\omega_o = 10^4 \qquad$ underdamped

$s_{1,2} = -8000 \pm j\sqrt{8000^2 - 10^8} = -8000 \pm j6000 \text{ rad/s}$

$i_{\rm L} = 60 \times 10^{-3} + B_1' e^{-8000t} \cos 6000t + B_2' e^{-8000t} \sin 6000t$

$-45 \times 10^{-3} = 60 \times 10^{-3} + B_1' \qquad \therefore \quad B_1' = -105\,\text{mA}$

$\dfrac{di_{\rm L}}{dt}(0) = -8000B_1' + 6000B_2' = 300$

$\therefore \quad B_2' = -90\,\text{mA}$

$i_{\rm L} = 60 - 105e^{-8000t} \cos 6000t - 90e^{-8000t} \sin 6000t\,\text{mA}, \quad t \geq 0$

P 8.31 $\alpha = \dfrac{1}{2RC} = \dfrac{1}{2(250)(0.2 \times 10^{-6})} = 10^4$

$\alpha^2 = 10^4 = \omega_o^2 \qquad$ critical damping

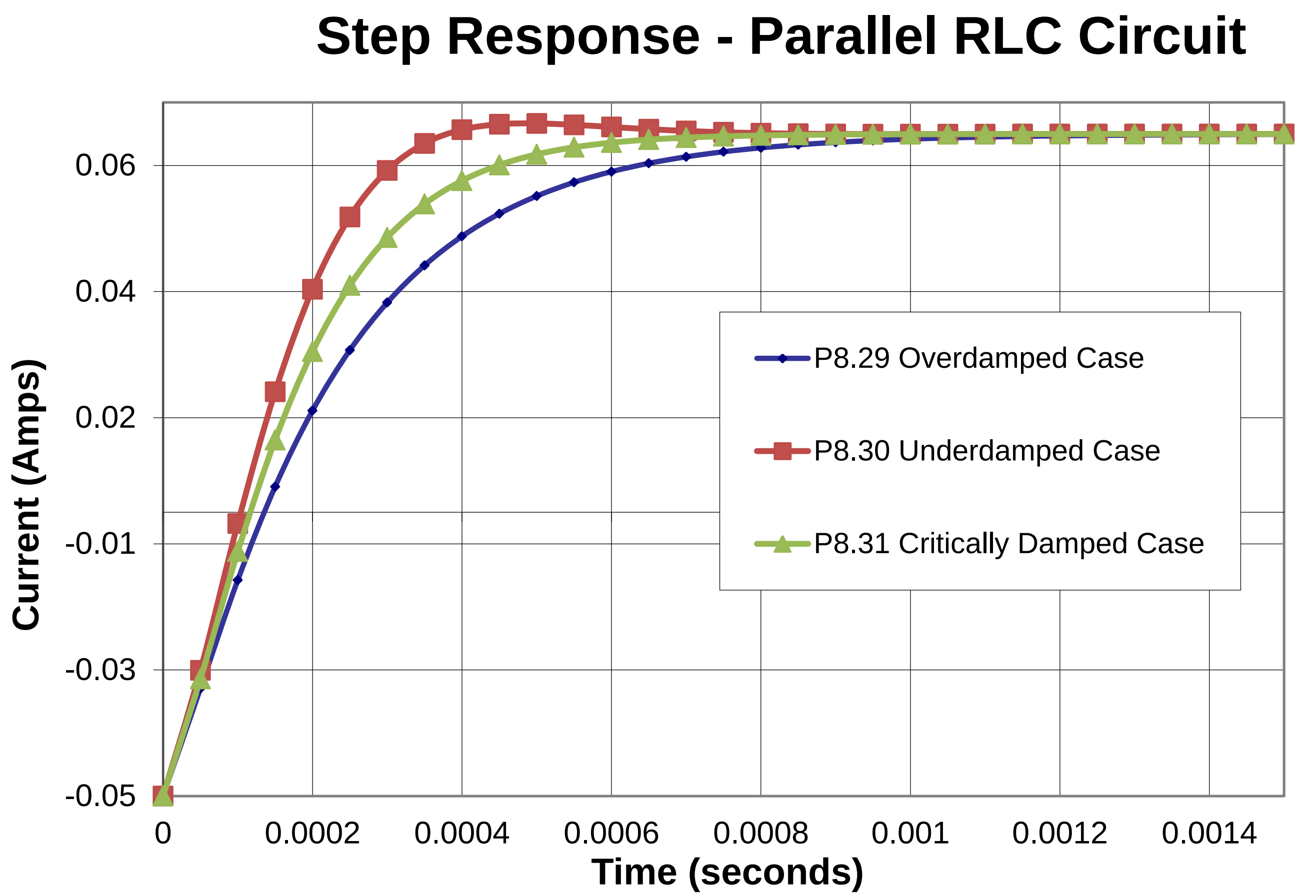
$i_{\rm L} = I_f + D_1' te^{-10^4 t} + D_2' e^{-10^4 t} = 60 \times 10^{-3} + D_1' te^{-10^4 t} + D_2' e^{-10^4 t}$

$i_{\rm L}(0) = -45 \times 10^{-3} = 60 \times 10^{-3} + D_2'; \qquad \therefore \quad D_2' = -105\,\text{mA}$

$\dfrac{di_{\rm L}}{dt}(0) = -10^4 D_2' + D_1' = 300\,\text{A/s}$

$\therefore \quad D_1' = 300 + 10^4(-105 \times 10^{-3}) = -750\,\text{A/s}$

$i_{\rm L} = 60 - 750{,}000te^{-10^4 t} - 105e^{-10^4 t}\,\text{mA}, \quad t \geq 0$



Step Response - Parallel RLC Circuit
Current (Amps)
0.06
0.04
0.02
-0.01
-0.03
-0.05
0
0.0002
0.0004
0.0006
0.0008
0.001
0.0012
0.0014
Time (seconds)
P8.29 Overdamped Case
P8.30 Underdamped Case
P8.31 Critically Damped Case

P 8.50 $\alpha = \dfrac{R}{2L} = 2000\,\text{rad/s}$

$$\omega_o^2 = \frac{1}{LC} = \frac{1}{(62.5\times10^{-3})(6.25\times10^{-6})} = 256\times10^4$$

$$s_{1,2} = -2000 \pm \sqrt{4\times10^6 - 256\times10^4} = -2000 \pm j1200\,\text{rad/s}$$

$$v_o = V_f + A_1' e^{-800t} + A_2' e^{-3200t}$$

$$v_o(0) = 0 = V_f + A_1' + A_2'$$

$$v_o(\infty) = 60\,\text{V}; \qquad \therefore \quad A_1' + A_2' = -60$$

$$\frac{dv_o(0)}{dt} = 0 = -800A_1' - 3200A_2'$$

$$\therefore \quad A_1' = -80\,\text{V}; \qquad\qquad A_2' = 20\,\text{V}$$

$$v_o = 60 - 80e^{-800t} + 20e^{-3200t}\,\text{V}, \quad t \geq 0$$