PROBLEM 13.11

13.11 The switch in the circuit in Fig. P13.11 has been closed for a long time before opening at $t = 0$.

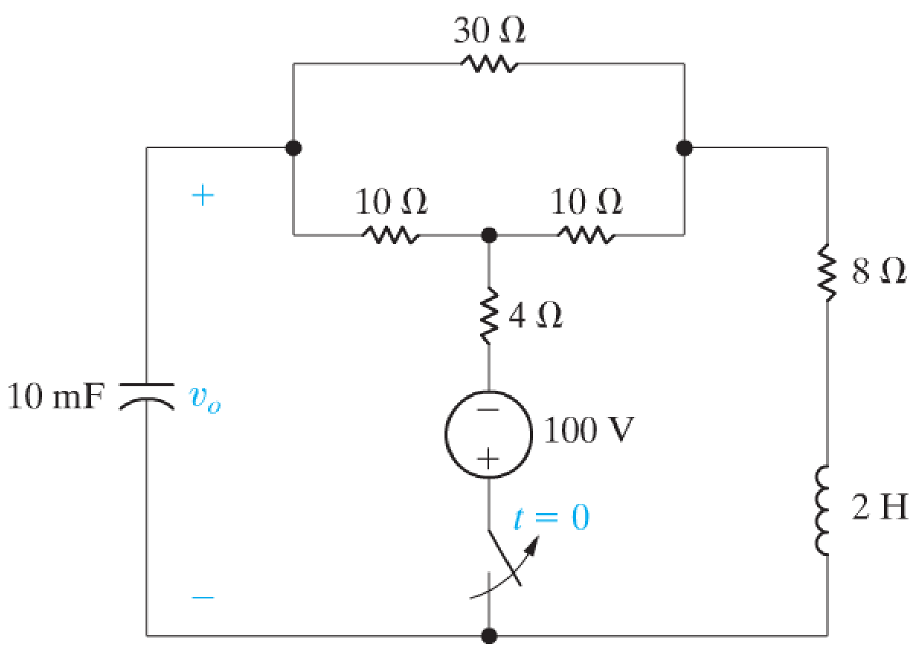
PSPICE
MULTISIM

a) Construct the s-domain equivalent circuit for $t > 0$.

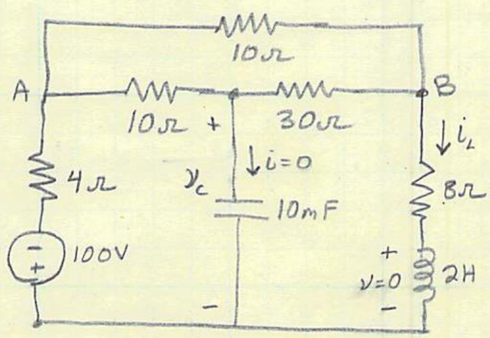
b) Find V_o.

c) Find v_o for $t \geq 0$.

Figure P13.11



CIRCUIT TO FIND INITIAL CONDITIONS



$$i_L = \frac{-100\text{V}}{8\Omega + 4\Omega + (10\Omega \| (10\Omega + 30\Omega))}$$

$$i_L = \frac{-100\text{V}}{12\Omega + \frac{400\Omega^2}{50\Omega}} = \frac{-100\text{V}}{20\Omega} = \underline{-5\text{A}}$$

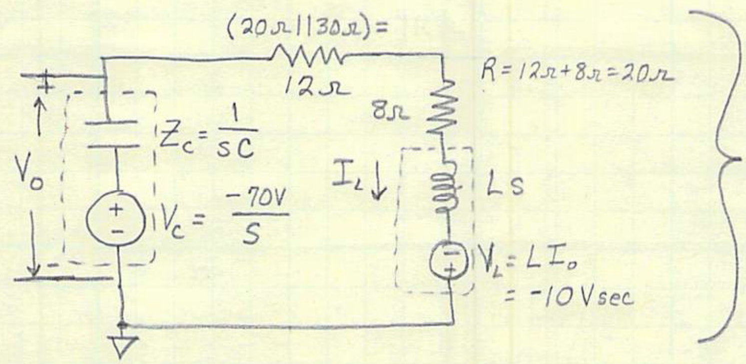
$$V_A = -100\text{V} - (-5\text{A})(4\Omega) = -80\text{V}$$

$$V_B = (-5\text{A})(8\Omega) = -40\text{V}$$

$$v_c = V_B + (V_A - V_B)\cdot\frac{30\Omega}{10\Omega + 30\Omega} = -40\text{V} + -40\text{V}\cdot\frac{3}{4}$$

$$v_c = -40\text{V} - 30\text{V} = \underline{-70\text{V}}$$

PROBLEM 13.11 (CONT'D)



(13.11a)

$$\text{NODAL:} \quad V_0\left(\frac{1}{Z_c} + \frac{1}{R+Ls}\right) - V_c\left(\frac{1}{Z_c}\right) - (-V_L)\frac{1}{(R+Ls)} = 0$$

$$V_0\left(Cs + \frac{1}{Ls+R}\right) = \frac{10\,\text{Vsec}}{(R+Ls)} - \frac{70V}{\cancel{s}}\cdot\cancel{s}C$$

$$V_0\left(\frac{LCs^2+RCs+1}{\cancel{Ls+R}}\right) = \frac{10\,\text{Vsec} - RC(70V) - LCs(70V)}{\cancel{(R+Ls)}}$$

$$V_0 = \frac{-[(LC\cdot 70V)\cdot s + (70V\cdot RC - 10V\cdot\text{sec})]}{(LCs^2+RCs+1)}$$

$$V_0 = \frac{-70V\cdot\cancel{LC}\left(s + \left(R/L - \frac{10V\cdot\text{sec}}{70V\cdot LC}\right)\right)}{\cancel{LC}\left(s^2 + (R/L)s + (1/LC)\right)}$$

$$R/L = \frac{20\Omega}{2H} = 10\,\text{r/sec} \quad \frac{1}{LC} = \frac{1}{2H\cdot 10mF} = 50\,\text{r}^2/\text{sec}^2$$

$$R/L - \frac{50}{7}\text{r/sec} = \left(\frac{70}{7} - \frac{50}{7}\right)\text{r/sec} = \frac{20}{7}\,\text{r/sec}$$

$$V_0 = -70V\,\frac{s + \left(\frac{20}{7}\,\text{r/sec}\right)}{s^2 + (10\,\text{r/sec})s + (50\,\text{r}^2/\text{sec}^2)} \qquad (13.11b)$$

$$s_0, s_1 = -\frac{10\,\text{r/sec}}{2} \pm \sqrt{25\,\text{r}^2/\text{sec}^2 - 50\,\text{r}^2/\text{sec}^2} = -5\,\text{r/sec}^2 \pm j5\,\text{r/sec} = -\alpha \pm j\beta$$

$$V_0 = -70V\left[\frac{K_1}{s + [5\,\text{r/sec}(1+j)]} + \frac{K_1^*}{s + [5\,\text{r/sec}(1-j)]}\right]$$

$$K_1 = \left.\frac{(s + \frac{20}{7}\,\text{r/sec})}{s + [5\,\text{r/sec}(1-j)]}\right|_{s=-(5\text{r/s}(1+j))} = \frac{-5\,\text{r/sec} + \frac{20}{7}\,\text{r/sec} - j5\,\text{r/sec}}{-j10\,\text{r/s}}$$

$$K_1 = \frac{1}{2}\left(1 - j\frac{3}{7}\right) = 0.5440\angle -156.80^\circ$$

$$V_0 = -70V\left[\frac{K_1}{s+(-s_1)} + \frac{K_1^*}{s+(-s_1^*)}\right] = -70V|K|\left[\frac{1\angle\theta_K}{s+(-s_1)} + \frac{1\angle-\theta_K}{s+(-s_1^*)}\right]$$

$$v(t) = -70V\cdot|K|\left[e^{j\theta_K}e^{-\alpha t}e^{-j\omega_0 t} + e^{-j\theta_K}e^{-\alpha t}e^{+j\omega_0 t}\right] = -70V\cdot|K|e^{-\alpha t}\cdot 2\cos(\omega t - \theta_K)$$

$$v(t) = -70V\cdot(0.5440)\cdot(2)\,e^{-5\text{r/s}\cdot t}\cos(5\,\text{r/s}\,t + 156.80^\circ)$$

$$v(t) = +76.2V\,e^{-\left(\frac{t}{200ms}\right)}\cos((5\,\text{r/s})t - 156.8^\circ) \qquad (13.11c)$$

CHECK: $v(t=0) \overset{?}{=} -70V = 76.2V\cos(-156.80^\circ) = 76.2V\cdot(-0.9191) = -70.0V$ ✓

PROBLEM # 13.36

13.36 PSPICE MULTISIM The magnetically coupled coils in the circuit seen in Fig. P13.36 carry initial currents of 15 and 10 A, as shown.

a) Find the initial energy stored in the circuit.

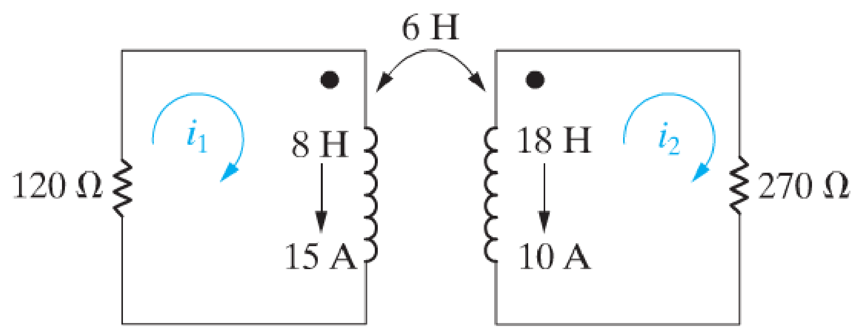
b) Find I_1 and I_2.

c) Find i_1 and i_2.

d) Find the total energy dissipated in the 120 and 270 Ω resistors.

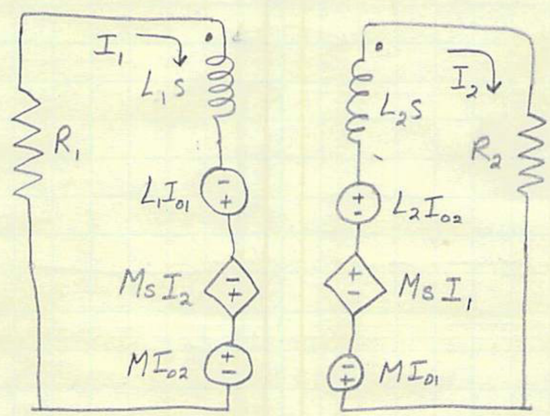
e) Repeat (a)–(d), with the dot on the 18 H inductor at the lower terminal.

Figure P13.36



IF FOR NO OTHER REASON, BECAUSE WE HAVE TO REPEAT EVERYTHING WITH THE DOTS MOVED, WE WILL WORK THE PROBLEM SYMBOLICALLY.

ALSO, IF $M=0$ WE KNOW THE ANSWERS BY INSPECTION SINCE EACH SIDE IS INDEPENDENT AND A SIMPLE LR FIRST-ORDER CIRCUIT. THIS PROVIDES AN OBVIOUS SANITY CHECK.



PROBLEM #13.36 (CONT'D)

$$U = \tfrac{1}{2}L_1 i_1^2 + \tfrac{1}{2}L_2 i_2^2 + M i_1 i_2 \qquad (\text{EQN } 6.66)$$

NOTE: i_1 & i_2 ARE THE CURRENTS ENTERING THE POLARITY MARKED TERMINAL.

$$U_{TOT} = \tfrac{1}{2}(8H)(15A)^2 + \tfrac{1}{2}(18H)(-10A)^2 - (6H)(15A)(-10A)$$

$$\therefore U_{TOT} = 2700\ J$$

$$\begin{aligned} I_1(R_1+L_1 s) - I_2(Ms) &= L_1 I_{01} - M I_{02} \\ -I_1(Ms) + I_2(R_2+L_2 s) &= L_2 I_{02} - M I_{01} \end{aligned}$$

NOTING THE SYMMETRY OF THE ABOVE EQUATIONS, WE CAN SOLVE FOR I_1 AND OBTAIN THE SOLUTION FOR I_2 BY SWAPPING '1' & '2' SUBSCRIPTS.

$$I_1\left[(R_1+L_1 s)(R_2+L_2 s) - (Ms)^2\right] = (L_1 I_{01} - M I_{02})(R_2+L_2 s) + (L_2 I_{02} - M I_{01})(Ms)$$

$$I_1 = \frac{s\left(L_1 I_{01} L_2 - \cancel{M L_2 I_{02}} + \cancel{M L_2 I_{02}} - M^2 I_{01}\right) + R_2(L_1 I_{01} - M I_{02})}{(L_1 L_2 - M^2)s^2 + (R_1 L_2 + R_2 L_1)s + R_1 R_2}$$

$$I_1 = I_{01}\frac{\left(s + \dfrac{R_2(L_1 I_{01} - M I_{02})}{I_{01}(L_1 L_2 - M^2)}\right)}{s^2 + \dfrac{(R_1 L_2 + R_2 L_1)}{(L_1 L_2 - M^2)}s + \dfrac{R_1 R_2}{(L_1 L_2 - M^2)}}$$

$$s_0, s_1 = \frac{1}{2}\left[\frac{-(R_1 L_2 + R_2 L_1)}{(L_1 L_2 - M^2)} \pm \sqrt{\frac{(R_1 L_2)^2 + R_1 R_2 L_1 L_2 + (R_2 L_1)^2}{(L_1 L_2 - M^2)^2} - \frac{4R_1 R_2(L_1 L_2 - M^2)}{(L_1 L_2 - M^2)^2}}\right]$$

$$s_0, s_1 = \frac{-(R_1 L_2 + R_2 L_1) \pm \sqrt{(R_1 L_2 - R_2 L_1)^2 + 4R_1 R_2 M^2}}{2(L_1 L_2 - M^2)}$$

CHECK: IF $M=0$: $s_0, s_1 = -\dfrac{R_1}{L_1}, -\dfrac{R_2}{L_2}$; $I_1 = I_{01}\dfrac{\cancel{(s + R_2/L_2)}}{(s + R_1/L_1)\cancel{(s + R_2/L_2)}}$ ✓

$$I_1 = I_{01}\frac{(s + z_1)}{(s - s_0)(s - s_1)}$$

FOR I_1: $z_1 = \dfrac{(270\Omega)[(8H)(15A) - (6H)(-10A)]}{(15A)[(8H)(18H) - (6H)^2]} = \dfrac{(270\Omega)(180\,HA)}{(15A)(108\,H^2)} = 30\ r/s$

$$\begin{aligned} (R_1 L_2 - R_2 L_1) &= (120\Omega)(18H) - (270\Omega)(8H) = 0 \\ (R_1 L_2 + R_2 L_1) &= (120\Omega)(18H) + (270\Omega)(8H) = 4320\,\Omega H \\ \sqrt{4R_1 R_2 M^2} &= 2(6H)\sqrt{(120\Omega)(270\Omega)} = 2160\,\Omega H \\ L_1 L_2 - M^2 &= (8H)(18H) - (6H)^2 = 108\,H^2 \end{aligned}$$

FOR I_1: $s_0, s_1 = \dfrac{-4320\,\Omega H \pm 2160\,\Omega H}{2 \cdot 108\,H^2} = (-20 \pm 10)\dfrac{\Omega}{H} = -30\ r/s, -10\ r/s$

FOR I_2: $z_1 = \dfrac{(120\Omega)[(18H)(-10A) - (6H)(15A)]}{(-10A)(108H^2)} = 30\ r/s$

s_0, s_1 = SAME AS FOR I_1

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PROBLEM # 13.36 (CONT'D)

$$I_1 = I_{o1}\frac{(S+Z_1)}{(S-S_0)(S-S_1)} = 15A\,\frac{(S+30^r/s)}{(S+30^r/s)(S+10^r/s)}$$

$$I_2 = I_{o2}\frac{S+Z_1}{(S-S_1)(S-S_2)} = -10A\,\frac{(S+30^r/s)}{(S+30^r/s)(S+10^r/s)}$$

$$I_1 = \frac{15A}{(S+10^r/s)}$$
$$I_2 = \frac{-10A}{(S+10^r/s)}$$
(#13.36b)

$$i_1(t) = 15A\,e^{-(10^r/s)t}u(t)$$
$$i_2(t) = -10A\,e^{-(10^r/s)t}u(t)$$
(#13.36c)

FOR AN EXPONENTIAL CURRENT THROUGH A RESISTOR:

$$U = \int_{-\infty}^{\infty} i^2(t)\cdot R\cdot dt = R\int_{-\infty}^{\infty}\left(I_o e^{-\alpha t}u(t)\right)^2 dt$$
$$= I_o^2 R\int_0^{\infty} e^{-2\alpha t}dt = I^2R\left(\frac{-1}{2\alpha}\right)e^{-2\alpha t}\Big|_0^{\infty}$$
$$U = \frac{I_o^2 R}{2\alpha}$$

$$U_{R1} = \frac{(15A)^2(120\Omega)}{2\cdot 10^r/s} = 1350\text{ J}$$
$$U_{R2} = \frac{(-10A)^2(270\Omega)}{2\cdot 10^r/s} = 1350\text{ J}$$
(#13.36d)

$$U_{TOT} = U_{R1} + U_{R2} = 2700\text{J} \checkmark$$

SWITCHING THE DOT LOCATION ON ONE INDUCTOR EFFECTIVELY MAKE $M \Rightarrow -M$.
TRACKING THIS CHANGE THROUGH THE PRIOR WORK YIELDS:

$$U_{TOT} = \tfrac{1}{2}(8H)(15A)^2 + \tfrac{1}{2}(18H)(-10A)^2 - (-6H)(15A)(-10A) = 900\text{J}$$ (a) (#13.36e)

$$I_1:\ Z_1 = \frac{(270\Omega)[(8H)(15A) - (-6H)(-10A)]}{(15A)(108H^2)} = 10^r/s$$

$$I_2:\ Z_1 = \frac{(120\Omega)[(18H)(-10A) - (-6H)(15A)]}{(-10A)(108H^2)} = 10^r/s$$

THE POLE LOCATIONS ARE UNCHANGED.

$$I_1 = \frac{15A}{(S+30^r/s)}\ ;\ I_2 = \frac{-10A}{(S+30^r/s)}$$ (b)

$$i_1(t) = 15A\,e^{-(30^r/s)t}u(t)\ ;\ i_2(t) = -10A\,e^{-(30^r/s)t}u(t)$$ (c)

$$U_1 = \frac{(15A)^2(120\Omega)}{2(30^r/s)} = 450\text{J};\ U_2 = \frac{(-10A)^2(270\Omega)}{2(30^r/s)} = 450\text{J}$$ (d)

$$U_T = U_1 + U_2 = 900\text{J} \checkmark$$