PROBLEM #1

A voltage signal, $v(t)$, is zero for all time less than t = 0. At t = 0 the voltage abruptly increases to 100V and begins to decay toward a final value of 20V with a time constant of 200 ms. Then, at t = 400 ms, the voltage abruptly changes to -50V and proceeds to decay toward zero with a time constant of 100 ms.

a) (1 pt) Accurately plot $v(t)$ from t = -100ms to t = 1s.

b) (1 pt) Accurately plot the derivative, $dv(t)/dt$, over this same time interval.

c) (1 pt) Write a single equation for $v(t)$, grouping terms by step function in ascending order of when the step function fires. In other words, your function should be of the form:

$$v(t) = (\ldots)u(t - T_0) + (\ldots)u(t - T_1) + (\ldots)u(t - T_2) + \cdots$$

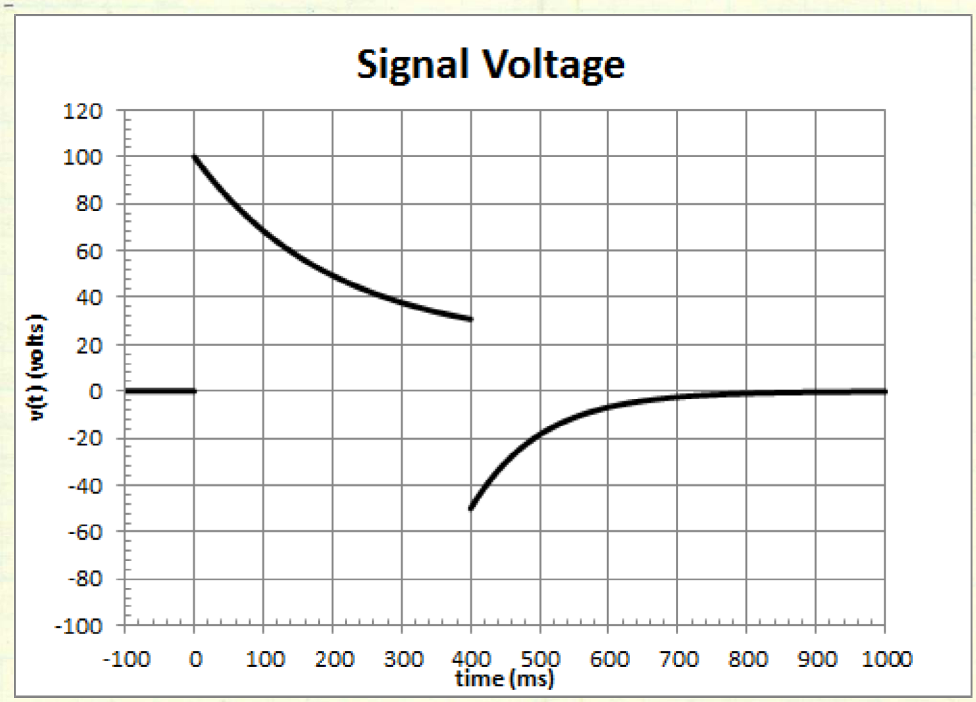
d) (1 pt) Similarly write a single equation for $dv(t)/dt$.
{}
e) (2 pt) Find $\mathscr{L}\{v(t)\}$?

f) (2 pt) Find $\mathscr{L}\{ dv(t)/dt \}$ starting from the result from part (d).

g) (2 pt) Find $\mathscr{L}\{ dv(t)/dt \}$ starting from the result from part (e).

$$v(t) \begin{cases} 0 & t \le 0 \\ 20V + (100V - 20V)\, e^{-t/200ms} & 0 < t < 400ms \\ -50V e^{-(t-T_0)/\tau_2} & 400\,ms < t \end{cases}$$

$$v(t) \begin{cases} 0 & t < 0 \\ V_1 + V_2\, e^{-t/\tau_1} & 0 < t < T_0 \\ V_3\, e^{-(t-T_0)/\tau_2} & T_0 < t \end{cases}$$



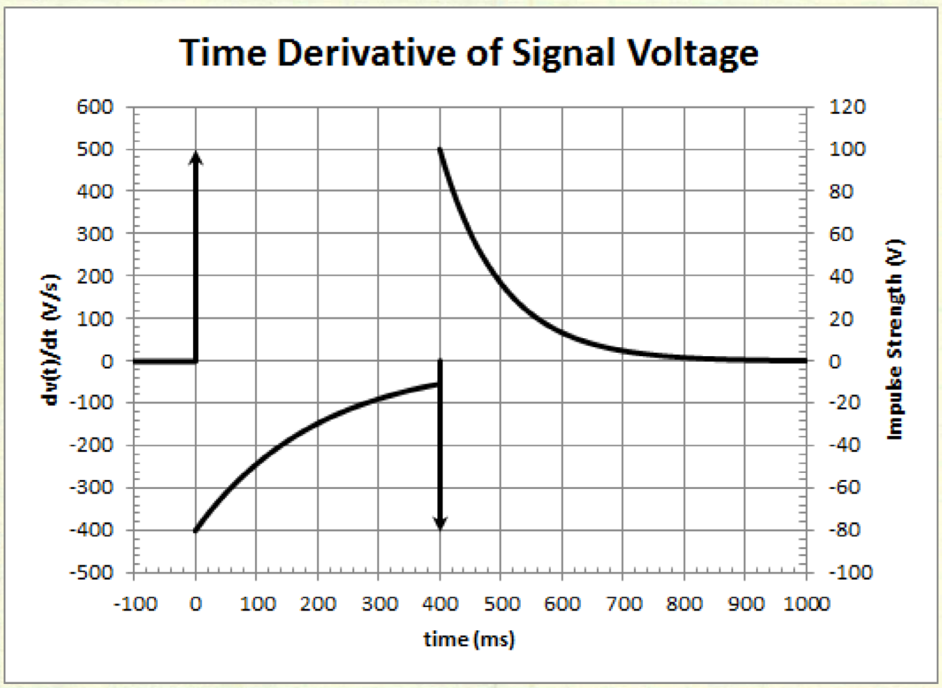
(#1a)

PROBLEM #1 (CONT'D)

$$\frac{d\nu(t)}{dt} = \begin{cases} 0 & t<0 \\ (V_1+V_0)\,\delta(t) & t=0 \\ -\frac{V_2}{\tau_1} e^{-\frac{t}{\tau_1}} & 0<t<T_0 \\ \left[V_3 - \left(V_1+V_2 e^{-\frac{T_0}{\tau_1}}\right)\right]\delta(t-T_0) & t=T_0 \\ -\frac{V_3}{\tau_2} e^{-\frac{t}{\tau_2}} & T_0<t \end{cases}$$

$$V_1+V_2 = 100\,V \;;\; \left[V_2 - \left(V_1+V_2 e^{-\frac{T_0}{\tau_1}}\right)\right] = -50\,V - \left(20V + 80\,V e^{-\frac{400ms}{200ms}}\right) = -80.83\,V$$

$$\frac{V_2}{\tau_1} = \frac{80V}{200ms} = 400\,V/s \;;\; \frac{V_3}{\tau_2} = \frac{-50V}{100ms} = -500\,V/s$$



(#1b)

$$\nu(t) = \left(V_1 + V_2 e^{-t/\tau_1}\right)u(t) + \left(V_3 e^{-(t-T_0)/\tau_2} - \left(V_1+V_2 e^{-t/\tau_1}\right)\right)u(t-T_0)$$

$$\underline{\underline{\nu(t) = \left(20V+80V e^{-\frac{t}{200ms}}\right)u(t) + \left(-50V e^{-\frac{(t-400ms)}{100ms}} - 20V - 80V e^{-\frac{t}{200ms}}\right)u(t-400ms)}} \leftarrow \text{(\#1c)}$$

$$\frac{d\nu(t)}{dt} = \left(V_1+V_2 e^{-t/\tau_1}\right)\delta(t) - \frac{V_2}{\tau_1} e^{-t/\tau_1} u(t) + \left(V_3 e^{-(t-T_0)/\tau_2} - \left(V_1+V_2 e^{-t/\tau_1}\right)\right)\delta(t) + \left(\frac{V_2}{\tau_1} e^{-t/\tau_1} - \frac{V_3}{\tau_2} e^{-\frac{(t-T_0)}{\tau_2}}\right)u(t-T_0)$$

$$\left.\left(V_1+V_2 e^{-\frac{t}{\tau_1}}\right)\right|_{t=0} = 20V+80V = 100V;$$

$$\left.\left(V_3 e^{-(t-T_0)/\tau_2} - \left(V_1+V_2 e^{-t/\tau_1}\right)\right)\right|_{t=T_0} = \left(-50V - \left(20V + 80V e^{-\frac{400ms}{200ms}}\right)\right) = -80.83\,V$$

$$\underline{\underline{\frac{d\nu(t)}{dt} = 100V\,\delta(t) - 400\,V/s\, e^{-\frac{t}{200ms}} u(t) - 80.8\,V\,\delta(t-400ms) + \left(400\,V/s\, e^{-t/200ms} + 500\,V/s\, e^{-(t-400ms)/100ms}\right)u(t-400ms)}} \leftarrow \text{(\#1d)}$$

PROBLEM #1 (CONT'D)

$$\mathcal{L}\{v(t)\} = \mathcal{L}\{V_1 u(t)\} \Rightarrow \frac{V_1}{s}$$

$$+\mathcal{L}\{V_2 e^{-\frac{t}{\tau_1}} u(t)\} \Rightarrow \frac{V_2}{(s+1/\tau_1)}$$

$$+\mathcal{L}\{V_3 e^{-\frac{(t-T_0)}{\tau_2}} u(t-T_0)\} \Rightarrow e^{-T_0 s}\frac{V_3}{(s+1/\tau_2)}$$

$$-\mathcal{L}\{V_1 u(t-T_0)\} \Rightarrow -e^{-T_0 s}\frac{V_1}{s}$$

$$-\mathcal{L}\{V_2 e^{-t/\tau_1} u(t-T_0)\} \Rightarrow -\mathcal{L}\{V_2 e^{-(t-T_0)/\tau_1} e^{-T_0/\tau_1} u(t-T_0)\} = -e^{-T_0 s}\frac{V_2 e^{-T_0/\tau_1}}{(s+1/\tau_1)}$$

ASIDE: $(V_2 e^{-T_0/\tau_1}) = 80V\, e^{-\frac{400ms}{200ms}} = 10.83V$

$$\mathcal{L}\{v(t)\} = \frac{20V}{s} + \frac{80V}{(s+1/200ms)} - e^{-(400ms)s}\left(\frac{20V}{s} + \frac{10.83V}{(s+1/200ms)} + \frac{50V}{(s+1/100ms)}\right) \leftarrow (\#1e)$$

$$\mathcal{L}\left\{\frac{dv(t)}{dt}\right\} = \mathcal{L}\{100V\,\delta(t)\} \Rightarrow 100V$$

$$-\mathcal{L}\{400V/s\; e^{-t/200ms}\} \Rightarrow -\frac{400\,V/sec}{(s+1/200ms)}$$

$$-\mathcal{L}\{80.83V\,\delta(t-400ms)\} \Rightarrow -80.83V\cdot e^{-(400ms)s}$$

$$+\mathcal{L}\{400V/s\; e^{-\frac{t}{200ms}} u(t-400ms)\} \Rightarrow \frac{(400V/sec)e^{-\frac{400ms}{200ms}}}{(s+1/200ms)} e^{-(400ms)s}$$

$$+\mathcal{L}\{500V/s\; e^{-\frac{(t-400ms)}{100ms}} u(t-400ms)\} \Rightarrow \frac{500\,V/sec}{(s+1/100ms)}\cdot e^{-(400ms)s}$$

ASIDE: $(400V/sec \cdot e^{-\frac{400ms}{200ms}}) = 54.13\,V/sec$

$$\mathcal{L}\left\{\frac{dv(t)}{dt}\right\} = 100V - \frac{400V/sec}{(s+1/200ms)} - e^{-(400ms)s}\left(80.83V - \frac{54.13V/sec}{(s+1/200ms)} - \frac{500V/sec}{(s+1/100ms)}\right) \leftarrow (\#1f)$$

$$\mathcal{L}\{v'(t)\} = s\mathcal{L}\{v(t)\} - \cancel{v(0)}^{\,0}$$

$$= 20V + \frac{(80V)s}{(s+1/200ms)} - e^{-(400ms)s}\left(20V + \frac{(10.83V)s}{(s+1/200ms)} + \frac{(50V)s}{(s+1/100ms)}\right)$$

ASIDE: $\frac{Ks}{(s+a)} = \frac{K(s+a-a)}{(s+a)} = \frac{K(s+a)}{(s+a)} - \frac{Ka}{(s+a)} = K - \frac{Ka}{(s+a)}$

$$= 20V + 80V - \frac{(80V)/200ms}{s+1/200ms} - e^{-(400ms)s}\left(20V + 10.83V - \frac{(10.83V)/200ms}{(s+1/200ms)} + 50V - \frac{(50V)/100ms}{(s+1/100ms)}\right)$$

$$\mathcal{L}\{v'(t)\} = 100V - \frac{400V/sec}{(s+1/200ms)} - e^{-(400ms)s}\left(80.83V - \frac{54.15V/sec}{(s+1/200ms)} - \frac{500V/sec}{(s+1/100ms)}\right) \leftarrow (\#1g)$$

$(\#1f) \stackrel{?}{=} (\#1g)$ ✓