PROBLEM #1

Scenario: You work for Pearson Education (the publisher of the textbook) as a Subject Matter Expert (SME). A reader has submitted the following feedback to the editor, who has then assigned you the task of clearing things up by indicating what, if any, corrections need to be made to either the textbook or MasteringEngineering and supporting your recommendation with both an "intuitive" explanation and a detailed technical verification of all of the material regarding the instantaneous power in three-phase circuits.

Dear Pearson,

The material described below from your textbook and the companion MasteringEngineering website appear to be either wrong or at least inconsistent. Please review and correct as needed.

In Section 11.5 of the textbook (p412), the author makes the claim that the total instantaneous power in a balanced three phase circuit is

$$p_T = 1.5V_m I_m \cos\theta_\varphi$$

They further make the claim that this is consistent with Eq. 11.35.

In the chapter summary, (p419), they state, "The total instantaneous power in a balanced three-phase circuit is constant and equals 1.5 times the average power per phase. (See page 412.)"

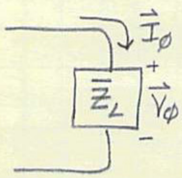
Finally, in the MasteringEngineering ME03 assignment, Part D of the Balanced Three-Phase Voltages tutorial offers as one of the possible answers to the question of the advantages of three-phase generation the following: "The total power available is constant and equal to three times the average power per phase." This is considered an incorrect answer on the basis that, "The total power available is equal to 1.5 times the average power per phase."

PROBLEM #1

SINCE ALL OF THE ISSUES RAISED CENTER AROUND THE TOTAL INSTANTANEOUS POWER AND THE AVERAGE PER PHASE POWER IN BALANCED THREE-PHASE CIRCUITS, WE BEGIN BY DERIVING THESE TWO QUANTITIES.

TOTAL INSTANTANEOUS POWER

LOOKING AT A SINGLE PHASE, SAY THE 'a' PHASE, THE INSTANTANEOUS POWER DELIVERED TO THE LOAD IS



$$p_A(t) = v_A(t)\, i_A(t)$$

$$p_A(t) = V_m I_m \cos(\omega t)\cos(\omega t - \theta_z)$$

SINCE THE CIRCUIT IN QUESTION IS BALANCED, THE OTHER TWO PHASES LEAD AND LAG THIS PHASE BY $\pm 120°$. CHOOSING A POSITIVE-SEQUENCE SYSTEM, WE THUS HAVE

① $p_A(t) = V_m I_m \cos(\omega t)\cos(\omega t - \theta_z)$

$$p_B(t) = V_m I_m \cos(\omega t - 120°)\cos(\omega t - \theta_z - 120°)$$

$$\cos(\omega t - 120°) = \cos(\omega t)\cos(120°) + \sin(\omega t)\sin(120°)$$
$$= -\tfrac{1}{2}\cos(\omega t) + \tfrac{\sqrt{3}}{2}\sin(\omega t)$$

$$\cos(\omega t - \theta_z - 120°) = \cos(\omega t - \theta_z)\cos(120°) + \sin(\omega t - \theta_z)\sin(120°)$$
$$= -\tfrac{1}{2}\cos(\omega t - \theta_z) + \tfrac{\sqrt{3}}{2}\sin(\omega t - \theta_z)$$

② $p_B(t) = V_m I_m \Big[\tfrac{1}{4}\cos(\omega t)\cos(\omega t - \theta_z) + \tfrac{3}{4}\sin(\omega t)\sin(\omega t - \theta_z) - \tfrac{\sqrt{3}}{4}\cos(\omega t)\sin(\omega t - \theta_z) - \tfrac{\sqrt{3}}{4}\sin(\omega t)\cos(\omega t - \theta_z)\Big]$

$$p_C(t) = V_m I_m \cos(\omega t + 120°)\cos(\omega t - \theta_z + 120°)$$

$$\cos(\omega t + 120°) = \cos(\omega t)\cos(120°) - \sin(\omega t)\sin(\omega t - \theta_z)$$
$$= -\tfrac{1}{2}\cos(\omega t) - \tfrac{\sqrt{3}}{2}\sin(\omega t)$$

$$\cos(\omega t - \theta_z + 120°) = \cos(\omega t - \theta_z)\cos(120°) - \sin(\omega t - \theta_z)\sin(120°)$$
$$= -\tfrac{1}{2}\cos(\omega t - \theta_z) - \tfrac{\sqrt{3}}{2}\sin(\omega t - \theta_z)$$

③ $p_C(t) = V_m I_m \Big[\tfrac{1}{4}\cos(\omega t)\cos(\omega t - \theta_z) + \tfrac{3}{4}\sin(\omega t)\sin(\omega t - \theta_z) + \tfrac{\sqrt{3}}{4}\cos(\omega t)\sin(\omega t - \theta_z) + \tfrac{\sqrt{3}}{4}\sin(\omega t)\cos(\omega t - \theta_z)\Big]$

$$p_T(t) = p_A(t) + p_B(t) + p_C(t) \qquad (① + ② + ③)$$
$$= V_m I_m \Big[\left(1 + \tfrac{1}{4} + \tfrac{1}{4}\right)\cos(\omega t)\cos(\omega t - \theta_z) + \left(\tfrac{3}{4} + \tfrac{3}{4}\right)\sin(\omega t)\sin(\omega t - \theta_z)$$
$$+ \left(-\tfrac{\sqrt{3}}{4} + \tfrac{\sqrt{3}}{4}\right)\cos(\omega t)\sin(\omega t - \theta_z) + \left(-\tfrac{\sqrt{3}}{4} + \tfrac{\sqrt{3}}{4}\right)\sin(\omega t)\cos(\omega t - \theta_z)\Big]$$
$$= 1.5 V_m I_m \left(\cos(\omega t)\cos(\omega t - \theta_z) + \sin(\omega t)\sin(\omega t - \theta_z)\right)$$
$$= 1.5 V_m I_m \cos(\omega t - (\omega t - \theta_z))$$

$$\therefore\ \underline{\underline{p_T(t) = 1.5\, V_m I_m \cos(\theta_z)}} \longleftarrow p_T(t)$$

PROBLEM #1 (CONT'D)

AVERAGE PER-PHASE POWER

BY SYMMETRY, THE AVERAGE POWER IN ALL PHASES ARE EQUAL. THEREFORE, WE CAN EXAMINE JUST THE A-PHASE AVERAGE POWER.

$$P_\phi = \frac{1}{T}\int_0^T p_A(t)\,dt$$

$$= \frac{1}{T}\int_0^T V_m I_m \cos(\omega t)\cos(\omega t - \theta_Z)\,dt$$

$$= \frac{1}{T} V_m I_m \int_0^T \cos(\omega t)\left[\cos(\omega t)\cos(\theta_Z) + \sin(\omega t)\sin(\theta_Z)\right] dt$$

$$= \frac{1}{T} V_m I_m \left[\int_0^T \cos^2(\omega t)\cos(\theta_Z)\,dt + \int_0^T \sin(\omega t)\cos(\omega t)\sin(\theta_Z)\,dt\right]$$

$$= \frac{1}{T} V_m I_m \left[\cos(\theta_Z)\int_0^T \cos(\omega t)\,dt + \sin(\theta_Z)\int_0^T \sin(\omega t)\cos(\omega t)\,dt\right]$$

$$= \frac{1}{T} V_m I_m \left[\cos(\theta_Z)\cdot\frac{T}{2} + \sin(\theta_Z)\cdot\frac{1}{2}\sin^2(\omega t)\Big|_0^{T=\frac{2\pi}{\omega}}\right]$$

$$= \frac{1}{T} V_m I_m \cos(\theta_Z)\frac{T}{2}$$

$$\therefore P_\phi = \frac{1}{2} V_m I_m \cos(\theta_Z) \longleftarrow P_\phi$$

WE CAN NOW ADDRESS THE CONCERNS RAISED. FIRST WE CAN CONFIRM THAT THE EQUATION GIVEN FOR THE TOTAL INSTANTANEOUS POWER NEAR THE BOTTOM OF P412 IN THE TEXT IS CORRECT.

EQN 11.35 STATES THAT

$$P_T = 3P_\phi = 3V_\phi I_\phi \cos\theta_\phi \quad \text{(EQN 11.35)}$$

FIRST, NOTE THAT $\theta_\phi = \theta_Z$ = THE LOAD IMPEDANCE ANGLE. SECOND, NOTE THAT V_m & I_m ARE THE AMPLITUDES OF SINUSOIDAL SIGNALS, WHILE V_ϕ & I_ϕ ARE, PER THE CONVENTIONS ESTABLISHED ON p405, RMS VALUES. THUS:

$$V_\phi = \frac{V_m}{\sqrt{2}}\ ;\ I_\phi = \frac{I_m}{\sqrt{2}}$$

THEREFORE, OUR RESULT FOR TOTAL INSTANTANEOUS POWER CAN BE WRITTEN AS

$$p_T(t) = 1.5(\sqrt{2}V_\phi)(\sqrt{2}I_\phi)\cos(\theta_\phi)$$

$$p_T(t) = 3V_\phi I_\phi \cos(\theta_\phi) = 3P_\phi$$

THUS, AS CLAIMED, THE TOTAL INSTANTANEOUS POWER IS CONSTANT AND IS CONSISTENT WITH EQN 11.35.

PROBLEM #1 (CONT'D)

THE CLAIM IN THE CHAPTER SUMMARY ON P.419 THAT THE TOTAL INSTANTANEOUS POWER IN A BALANCED THREE-PHASE CIRCUIT IS CONSTANT AND EQUALS 1.5 TIMES THE AVERAGE POWER PER PHASE IS ONLY PARTLY CORRECT. SINCE THE TOTAL INSTANTANEOUS POWER HAS NO TIME-DEPENDENCE, IT IS TRUE THAT IT IS CONSTANT. HOWEVER, IT IS EQUAL TO 3 TIMES THE AVERAGE POWER PER PHASE. NOT ONLY IS THIS BORN OUT BY THE DERIVATION ABOVE, BUT IT IS REQUIRED BY THE CONSERVATION OF ENERGY. IF THE TOTAL POWER DELIVERED IS CONSTANT, THEN IT MUST BE EQUAL TO THE AVERAGE TOTAL POWER DELIVERED. IF EACH PHASE CONSUMES POWER SYMMETRICALLY, THE THIS MUST IN TURN BE EQUAL TO THREE TIMES THE AVERAGE PER-PHASE POWER.

THE PROBLEM IN MASTERING ENGINEERING IS WRONG IN TWO RESPECTS. FIRST, THE OFFERED ANSWER IS CORRECT AND SHOULD NOT BE CONSIDERED INCORRECT. SECOND, THE REASON GIVEN JUSTIFYING THE CLAIM THAT THE ANSWER IS INCORRECT SUFFERS FROM THE SAME ERROR THAT THE CLAIM IN THE SUMMARY DOES.

THE MOST LIKELY REASON FOR THE ERRONEOUS CLAIMS IS THE FAILURE TO RECOGNIZE THAT

$$P_\phi \neq V_m I_m \cos(\theta_\phi)$$

BECAUSE V_m AND I_m ARE NOT RMS VALUES. INSTEAD

$$P_\phi = \frac{V_m I_m}{2} \cos(\theta_\phi)$$

THEREFORE

$$\begin{aligned} p_T(t) &= 1.5\, V_m I_m \cos(\theta_\phi) \\ &= 1.5 \cdot 2 \cdot \frac{V_m I_m}{2} \cos(\theta_\phi) \end{aligned}$$

$$\underline{p_T(t) = 3\, P_\phi}$$

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