EENG382 HW09 – AUTHOR'S SOLUTIONS

NOTE: I have not yet verified that the author's solutions are, in fact, correct.

Prob 16.17

P 16.17 **[a]** $i(t)$ is odd, therefore $a_v = 0$ and $b_k = 0$ for all k.

$$f(t) = i(t) = I_m - \frac{2I_m}{T}t, \quad 0 \leq t \leq T$$

$$b_k = \frac{4}{T}\int_0^{T/2} f(t)\sin k\omega_o t\,dt$$

$$= \frac{4}{T}\int_0^{T/2}\left(I_m - \frac{2I_m}{T}t\right)\sin k\omega_o t\,dt$$

$$= \frac{4I_m}{T}\left[\int_0^{T/2}\sin k\omega_0 t\,dt - \frac{2}{T}\int_0^{T/2} t\sin k\omega_0 t\,dt\right]$$

$$= \frac{4I_m}{T}\left[\frac{-\cos k\omega_0 t}{k\omega_0}\Big|_0^{T/2} - \frac{2}{T}\left(\frac{\sin k\omega_0 t}{k^2\omega_0^2} - \frac{t\cos k\omega_0 t}{k\omega_0}\right)\Big|_0^{T/2}\right]$$

$$= \frac{4I_m}{T}\left[\frac{1-\cos k\pi}{k\omega_0} + \frac{\cos k\pi}{k\omega_0}\right]$$

$$= \frac{4I_m}{k\omega_0 T} = \frac{2I_m}{k\pi}$$

$$\therefore \quad i(t) = \frac{2I_m}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin n\omega_o t$$

[b] $$i(t) = \frac{2I_m}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin n\omega_o(t + T/2)$$

$$= \frac{2I_m}{\pi}\sum_{n=1}^{\infty}\frac{\cos n\pi}{n}\sin n\omega_o t$$

Prob 16.29

P 16.29 [**a**] $$\frac{V_0 - V_g}{16s} + V_0(12.5 \times 10^{-6}s) + \frac{V_0}{1000} = 0$$

$$V_0\left[\frac{1}{16s} + 12.6 \times 10^{-6}s + \frac{1}{1000}\right] = \frac{V_g}{16s}$$

$$V_0(1000 + 0.2s^2 + 16s) = 1000V_g$$

$$V_0 = \frac{5000V_g}{s^2 + 80s + 5000}$$

$$I_0 = \frac{V_0}{1000} = \frac{5V_g}{s^2 + 80s + 5000}$$

$$H(s) = \frac{I_0}{V_g} = \frac{5}{s^2 + 80s + 5000}$$

$$H(nj\omega_0) = \frac{5}{(5000 - n^2\omega_0^2) + j80n\omega_0}$$

$$\omega_0 = \frac{2\pi}{T} = 240\pi; \qquad \omega_0^2 = 57{,}600\pi^2; \qquad 80\omega_0 = 19{,}200\pi$$

$$H(jn\omega_0) = \frac{5}{(5000 - 57{,}600\pi^2n^2) + j19{,}200\pi n}$$

$$H(0) = 10^{-3}$$

$$H(j\omega_0) = 8.82 \times 10^{-6}\underline{/-173.89^\circ}$$

$$H(j2\omega_0) = 2.20 \times 10^{-6}\underline{/-176.96^\circ}$$

$$H(j3\omega_0) = 9.78 \times 10^{-7}\underline{/-177.97^\circ}$$

$$H(j4\omega_0) = 5.5 \times 10^{-7}\underline{/-178.48^\circ}$$

Prob 16.29 (Cont'd)

$$v_g = \frac{680}{\pi} - \frac{1360}{\pi}\left[\frac{1}{3}\cos\omega_0 t + \frac{1}{15}\cos 2\omega_0 t + \frac{1}{35}\cos 3\omega_0 t + \frac{1}{63}\cos 4\omega_0 t + \dots\right]$$

$$i_0 = \frac{680}{\pi}\times 10^{-3} - \frac{1360}{3\pi}(8.82\times 10^{-6})\cos(\omega_0 t - 173.89^\circ)$$

$$-\frac{1360}{15\pi}(2.20\times 10^{-6})\cos(2\omega_0 t - 176.96^\circ)$$

$$-\frac{1360}{35\pi}(9.78\times 10^{-7})\cos(3\omega_0 t - 177.97^\circ)$$

$$-\frac{1360}{63\pi}(5.5\times 10^{-7})\cos(4\omega_0 t - 178.48^\circ) - \dots$$

$$= 216.45\times 10^{-3} + 1.27\times 10^{-3}\cos(240\pi t + 6.11^\circ)$$

$$+6.35\times 10^{-5}\cos(480\pi t + 3.04^\circ)$$

$$+1.21\times 10^{-5}\cos(720\pi t + 2.03^\circ)$$

$$+3.8\times 10^{-6}\cos(960\pi t + 1.11^\circ) - \dots$$

$$i_0 \cong 216.45 + 1.27\cos(240\pi t + 6.11^\circ)\,\text{mA}$$

Note that the sinusoidal component is very small compared to the dc component, so

$i_0 \cong 216.45\,\text{mA}$ (a dc current)

[b] The circuit is a low pass filter, so the harmonic terms are greatly reduced in the output.

Prob 16.34

P 16.34 **[a]** $a_v = \dfrac{2\left(\frac{1}{2}\,\frac{T}{4}V_m\right)}{T} = \dfrac{V_m}{4}$

$$a_k = \frac{4}{T}\int_0^{T/4}\left[V_m - \frac{4V_m}{T}t\right]\cos k\omega_o t\,dt$$

$$= \frac{4V_m}{\pi^2 k^2}\left[1 - \cos\frac{k\pi}{2}\right]$$

$$b_k = 0, \quad \text{all } k$$

$$a_v = \frac{60}{4} = 15\,\text{V}$$

$$a_1 = \frac{240}{\pi^2}$$

$$a_2 = \frac{240}{4\pi^2}(1 - \cos\pi) = \frac{120}{\pi^2}$$

$$V_{\text{rms}} = \sqrt{(15)^2 + \frac{1}{2}\left[\left(\frac{240}{\pi^2}\right)^2 + \left(\frac{120}{\pi^2}\right)^2\right]} = 24.38\,\text{V}$$

$$P = \frac{(24.38)^2}{10} = 59.46\,\text{W}$$

[b] Area under v^2; $\qquad 0 \le t \le T/4$

$$v^2 = 3600 - \frac{28{,}800}{T}t + \frac{57{,}600}{T^2}t^2$$

$$A = 2\int_0^{T/4}\left[3600 - \frac{28{,}800}{T}t + \frac{57{,}600}{T^2}t^2\right]\,dt = 600T$$

$$V_{\text{rms}} = \sqrt{\frac{1}{T}600T} = \sqrt{600} = 24.49\,\text{V}$$

$$P = \sqrt{600}^2/10 = 60\,\text{W}$$

[c] Error $= \left(\dfrac{59.46}{60.00} - 1\right)100 = -0.9041\%$