EENG382 HW08 – AUTHOR'S SOLUTIONS

NOTE: I have not yet verified that the author's solutions are, in fact, correct.

Prob 15.14

P 15.14 $H(s) = \dfrac{(R/L)s}{s^2 + (R/L)s + (1/LC)} = \dfrac{\beta s}{s^2 + \beta s + \omega_o^2}$

For the prototype circuit $\omega_o = 1$ and $\beta = \omega_o/Q = 1/Q$.
For the scaled circuit

$$H'(s) = \frac{(R'/L')s}{s^2 + (R'/L')s + (1/L'C')}$$

where $R' = k_m R$; $L' = \dfrac{k_m}{k_f}L$; and $C' = \dfrac{C}{k_f k_m}$

$$\therefore \quad \frac{R'}{L'} = \frac{k_m R}{\frac{k_m}{k_f}L} = k_f\left(\frac{R}{L}\right) = k_f \beta$$

$$\frac{1}{L'C'} = \frac{k_f k_m}{\frac{k_m}{k_f}LC} = \frac{k_f^2}{LC} = k_f^2$$

$$Q' = \frac{\omega_o'}{\beta'} = \frac{k_f \omega_o}{k_f \beta} = Q$$

therefore the Q of the scaled circuit is the same as the Q of the unscaled circuit. Also note $\beta' = k_f \beta$.

$$\therefore \quad H'(s) = \frac{\left(\frac{k_f}{Q}\right)s}{s^2 + \left(\frac{k_f}{Q}\right)s + k_f^2}$$

$$H'(s) = \frac{\left(\frac{1}{Q}\right)\left(\frac{s}{k_f}\right)}{\left[\left(\frac{s}{k_f}\right)^2 + \frac{1}{Q}\left(\frac{s}{k_f}\right) + 1\right]}$$

Prob 15.45

P 15.45 From Eq 15.56 we can write

$$H(s) = \frac{-\left(\frac{2}{R_3C}\right)\left(\frac{R_3C}{2}\right)\left(\frac{1}{R_1C}\right)s}{s^2 + \frac{2}{R_3C}s + \frac{R_1+R_2}{R_1R_2R_3C^2}}$$

or

$$H(s) = \frac{-\left(\frac{R_3}{2R_1}\right)\left(\frac{2}{R_3C}s\right)}{s^2 + \frac{2}{R_3C}s + \frac{R_1+R_2}{R_1R_2R_3C^2}}$$

Therefore

$$\frac{2}{R_3C} = \beta = \frac{\omega_o}{Q}; \qquad \frac{R_1+R_2}{R_1R_2R_3C^2} = \omega_o^2;$$

and $K = \frac{R_3}{2R_1}$

By hypothesis $C = 1\,\text{F}$ and $\omega_o = 1\,\text{rad/s}$

$$\therefore \quad \frac{2}{R_3} = \frac{1}{Q} \text{ or } R_3 = 2Q$$

$$R_1 = \frac{R_3}{2K} = \frac{Q}{K}$$

$$\frac{R_1+R_2}{R_1R_2R_3} = 1$$

$$\frac{Q}{K} + R_2 = \left(\frac{Q}{K}\right)(2Q)R_2$$

$$\therefore \quad R_2 = \frac{Q}{2Q^2 - K}$$

Prob 15.50

P 15.50 [a] At very low frequencies the two capacitor branches are open and because the op amp is ideal the current in R_3 is zero. Therefore at low frequencies the circuit behaves as an inverting amplifier with a gain of R_2/R_1. At very high frequencies the capacitor branches are short circuits and hence the output voltage is zero.

[b] Let the node where R_1, R_2, R_3, and C_2 join be denoted as a, then

$$(V_a - V_i)G_1 + V_a sC_2 + (V_a - V_o)G_2 + V_a G_3 = 0$$

$$-V_a G_3 - V_o sC_1 = 0$$

or

$$(G_1 + G_2 + G_3 + sC_2)V_a - G_2 V_o = G_1 V_i$$

$$V_a = \frac{-sC_1}{G_3} V_o$$

Solving for V_o/V_i yields

$$H(s) = \frac{-G_1 G_3}{(G_1 + G_2 + G_3 + sC_2)sC_1 + G_2 G_3}$$

$$= \frac{-G_1 G_3}{s^2 C_1 C_2 + (G_1 + G_2 + G_3)C_1 s + G_2 G_3}$$

$$= \frac{-G_1 G_3 / C_1 C_2}{s^2 + \left[\frac{(G_1+G_2+G_3)}{C_2}\right] s + \frac{G_2 G_3}{C_1 C_2}}$$

$$= \frac{-\frac{G_1 G_2 G_3}{G_2 C_1 C_2}}{s^2 + \left[\frac{(G_1+G_2+G_3)}{C_2}\right] s + \frac{G_2 G_3}{C_1 C_2}}$$

$$= \frac{-K b_o}{s^2 + b_1 s + b_o}$$

where $K = \frac{G_1}{G_2}$; $\qquad b_o = \frac{G_2 G_3}{C_1 C_2}$

and $b_1 = \frac{G_1 + G_2 + G_3}{C_2}$

Prob 15.50 (Cont'd)

[**c**] Rearranging we see that

$$G_1 = KG_2$$

$$G_3 = \frac{b_oC_1C_2}{G_2} = \frac{b_oC_1}{G_2}$$

since by hypothesis $C_2 = 1\,\mathrm{F}$

$$b_1 = \frac{G_1 + G_2 + G_3}{C_2} = G_1 + G_2 + G_3$$

$$\therefore\ b_1 = KG_2 + G_2 + \frac{b_oC_1}{G_2}$$

$$b_1 = G_2(1 + K) + \frac{b_oC_1}{G_2}$$

Solving this quadratic equation for G_2 we get

$$G_2 = \frac{b_1}{2(1+K)} \pm \sqrt{\frac{b_1^2 - b_oC_1 4(1+K)}{4(1+K)^2}}$$

$$= \frac{b_1 \pm \sqrt{b_1^2 - 4b_o(1+K)C_1}}{2(1+K)}$$

For G_2 to be realizable

$$C_1 < \frac{b_1^2}{4b_o(1+K)}$$

[**d**] 1. Select $C_2 = 1\,\mathrm{F}$

2. Select C_1 such that $C_1 < \dfrac{b_1^2}{4b_o(1+K)}$

3. Calculate G_2 (R_2)

4. Calculate G_1 (R_1); $G_1 = KG_2$

5. Calculate G_3 (R_3); $G_3 = b_oC_1/G_2$