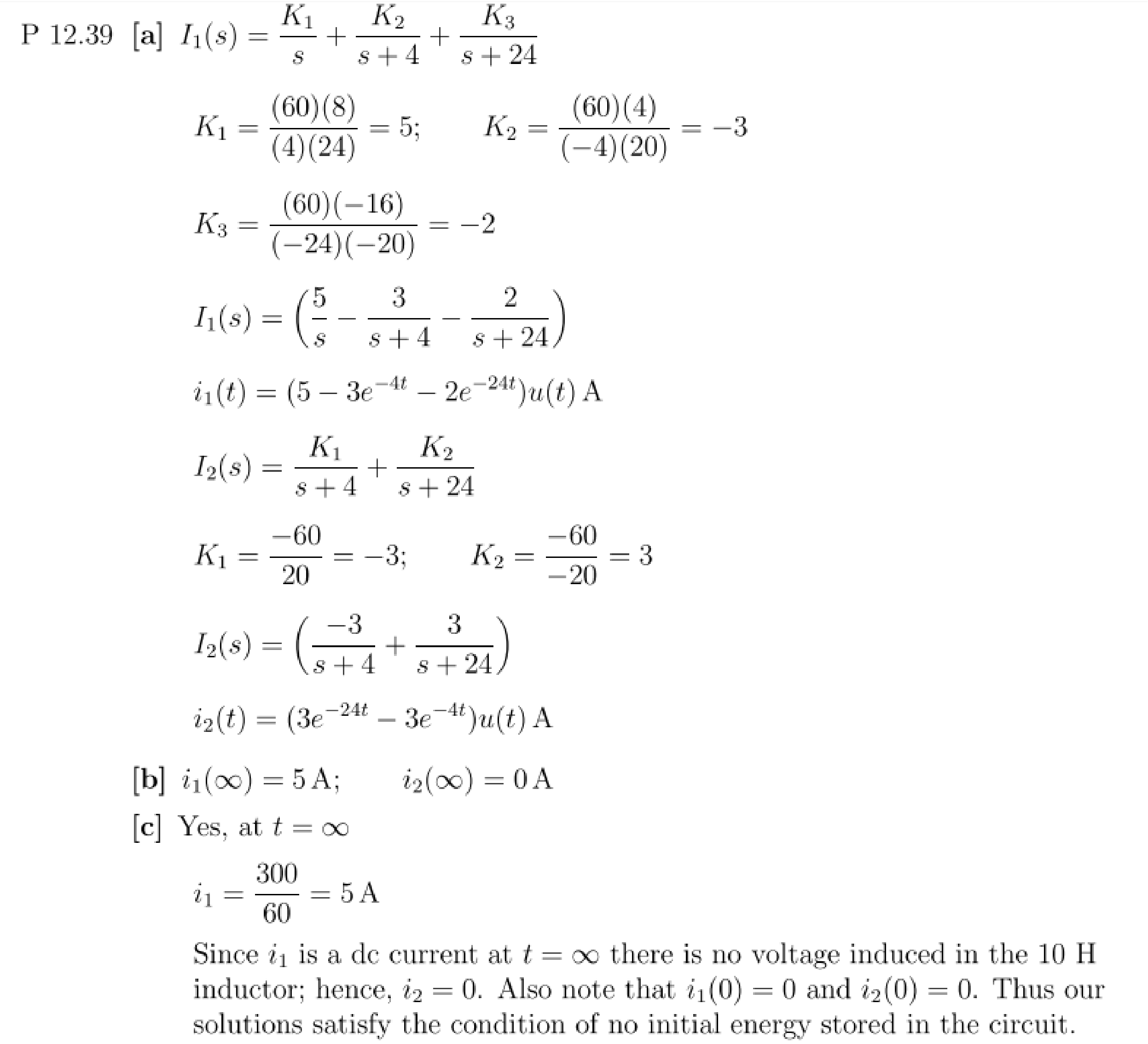
EENG382 HW04 – AUTHOR'S SOLUTIONS

NOTE: I have not yet verified that the author's solutions are, in fact, correct.

Prob 12.39



P 12.39 **[a]** $I_1(s) = \frac{K_1}{s} + \frac{K_2}{s+4} + \frac{K_3}{s+24}$

$$K_1 = \frac{(60)(8)}{(4)(24)} = 5; \qquad K_2 = \frac{(60)(4)}{(-4)(20)} = -3$$

$$K_3 = \frac{(60)(-16)}{(-24)(-20)} = -2$$

$$I_1(s) = \left(\frac{5}{s} - \frac{3}{s+4} - \frac{2}{s+24}\right)$$

$$i_1(t) = (5 - 3e^{-4t} - 2e^{-24t})u(t)\,\text{A}$$

$$I_2(s) = \frac{K_1}{s+4} + \frac{K_2}{s+24}$$

$$K_1 = \frac{-60}{20} = -3; \qquad K_2 = \frac{-60}{-20} = 3$$

$$I_2(s) = \left(\frac{-3}{s+4} + \frac{3}{s+24}\right)$$

$$i_2(t) = (3e^{-24t} - 3e^{-4t})u(t)\,\text{A}$$

[b] $i_1(\infty) = 5\,\text{A}; \qquad i_2(\infty) = 0\,\text{A}$

[c] Yes, at $t = \infty$

$$i_1 = \frac{300}{60} = 5\,\text{A}$$

Since i_1 is a dc current at $t = \infty$ there is no voltage induced in the 10 H inductor; hence, $i_2 = 0$. Also note that $i_1(0) = 0$ and $i_2(0) = 0$. Thus our solutions satisfy the condition of no initial energy stored in the circuit.

Prob 12.42

P 12.42 [a]

$$\begin{array}{rr} & 5 \\ \cline{2-2} F(s) = \quad s^2+6s+8 \big) & 5s^2+38s+80 \\ & 5s^2+30s+40 \\ \cline{2-2} & 8s+40 \end{array}$$

$$F(s) = 5 + \frac{8s+40}{s^2+6s+8} = 10 + \frac{K_1}{s+2} + \frac{K_2}{s+4}$$

$$K_1 = \frac{8s+40}{s+4}\bigg|_{s=-2} = 12$$

$$K_2 = \frac{8s+40}{s+2}\bigg|_{s=-4} = -4$$

$$f(t) = 5\delta(t) + [12e^{-2t} - 4e^{-4t}]u(t)$$

[b]

$$\begin{array}{rr} & 10 \\ \cline{2-2} F(s) = \quad s^2+48s+625 \big) & 10s^2+512s+7186 \\ & 10s^2+480s+6250 \\ \cline{2-2} & 32s+936 \end{array}$$

$$F(s) = 10 + \frac{32s+936}{s^2+48s+625} = 10 + \frac{K_1}{s+24-j7} + \frac{K_2^*}{s+24+j7}$$

$$K_1 = \frac{32s+936}{s+24+j7}\bigg|_{s=-24+j7} = 16 - j12 = 20\underline{/-36.87^\circ}$$

$$f(t) = 10\delta(t) + [40e^{-24t}\cos(7t - 36.87^\circ)]u(t)$$

[c]

$$\begin{array}{rr} & s-10 \\ \cline{2-2} F(s) = \quad s^2+15s+50 \big) & s^3+5s^2-50s-100 \\ & s^3+15s^2+50s \\ \cline{2-2} & -10s^2-100s-100 \\ & -10s^2-150s-500 \\ \cline{2-2} & 50s+400 \end{array}$$

$$F(s) = s - 10 + \frac{K_1}{s+5} + \frac{K_2}{s+10}$$

$$K_1 = \frac{50s+400}{s+10}\bigg|_{s=-5} = 30$$

$$K_2 = \frac{50s+400}{s+5}\bigg|_{s=-10} = 20$$

$$f(t) = \delta'(t) - 10\delta(t) + [30e^{-5t} + 20e^{-10t}]u(t)$$

Prob 12.43

P 12.43 **[a]** $F(s) = \dfrac{K_1}{s^2} + \dfrac{K_2}{s} + \dfrac{K_3}{s+1-j2} + \dfrac{K_3^*}{s+1+j2}$

$$K_1 = \frac{100(s+1)}{s^2+2s+5}\bigg|_{s=0} = 20$$

$$K_2 = \frac{d}{ds}\left[\frac{100(s+1)}{s^2+2s+5}\right] = \left[\frac{100}{s^2+2s+5} - \frac{100(s+1)(2s+2)}{(s^2+2s+5)^2}\right]_{s=0}$$

$$= 20 - 8 = 12$$

$$K_3 = \frac{100(s+1)}{s^2(s+1+j2)}\bigg|_{s=-1+j2} = -6 + j8 = 10\underline{/126.87^\circ}$$

$$f(t) = [20t + 12 + 20e^{-t}\cos(2t + 126.87^\circ)]u(t)$$

[b] $F(s) = \dfrac{K_1}{s} + \dfrac{K_2}{(s+5)^3} + \dfrac{K_3}{(s+5)^2} + \dfrac{K_4}{s+5}$

$$K_1 = \frac{500}{(s+5)^3}\bigg|_{s=0} = 4$$

$$K_2 = \frac{500}{s}\bigg|_{s=-5} = -100$$

$$K_3 = \frac{d}{ds}\left[\frac{500}{s}\right] = \frac{-500}{s^2}\bigg|_{s=-5} = -20$$

$$K_4 = \frac{1}{2}\frac{d}{ds}\left[\frac{-500}{s^2}\right] = \frac{1}{2}\frac{1000}{s^3}\bigg|_{s=-5} = -4$$

$$f(t) = [4 - 50t^2e^{-5t} - 20te^{-5t} - 4e^{-5t}]u(t)$$

[c] $F(s) = \dfrac{K_1}{s} + \dfrac{K_2}{(s+1)^3} + \dfrac{K_3}{(s+1)^2} + \dfrac{K_4}{s+1}$

$$K_1 = \frac{40(s+2)}{(s+1)^3}\bigg|_{s=0} = 80$$

$$K_2 = \frac{40(s+2)}{s}\bigg|_{s=-1} = -40$$

$$K_3 = \frac{d}{ds}\left[\frac{40(s+2)}{s}\right] = \left[\frac{40}{s} - \frac{40(s+2)}{s^2}\right]_{s=-1} = -40 - 40 = -80$$

$$K_4 = \frac{1}{2}\frac{d}{ds}\left[\frac{40}{s} - \frac{40(s+2)}{s^2}\right]$$

$$= \frac{1}{2}\left[\frac{-40}{s^2} - \frac{40}{s^2} + \frac{80(s+2)}{s^3}\right]_{s=-1} = \frac{1}{2}(-40 - 40 - 80) = -80$$

$$f(t) = [80 - 20t^2e^{-t} - 80te^{-t} - 80e^{-t}]u(t)$$

Prob 12.43 (Cont’d)

[d] $F(s) = \frac{K_1}{s} + \frac{K_2}{(s+1)^4} + \frac{K_3}{(s+1)^3} + \frac{K_4}{(s+1)^2} + \frac{K_5}{s+1}$

$$K_1 = \frac{(s+5)^2}{(s+1)^4}\bigg|_{s=0} = 25$$

$$K_2 = \frac{(s+5)^2}{s}\bigg|_{s=-1} = -16$$

$$K_3 = \frac{d}{ds}\left[\frac{(s+5)^2}{s}\right] = \left[\frac{2(s+5)}{s} - \frac{(s+5)^2}{s^2}\right]_{s=-1}$$

$$= -8 - 16 = -24$$

$$K_4 = \frac{1}{2}\frac{d}{ds}\left[\frac{2(s+5)}{s} - \frac{(s+5)^2}{s^2}\right]$$

$$= \frac{1}{2}\left[\frac{2}{s} - \frac{2(s+5)}{s^2} - \frac{2(s+5)}{s^2} + \frac{3(s+5)^2}{s^3}\right]_{s=-1}$$

$$= \frac{1}{2}(-2 - 8 - 8 - 32) = -25$$

$$K_5 = \frac{1}{6}\frac{d}{ds}\left[\frac{2}{s} - \frac{2(s+5)}{s^2} - \frac{2(s+5)}{s^2} + \frac{3(s+5)^2}{s^3}\right]$$

$$= \frac{1}{6}\left[\frac{-2}{s^2} - \frac{2}{s^2} + \frac{4(s+5)}{s^3} - \frac{2}{s^2} + \frac{4(s+5)}{s^3} + \frac{4(s+5)}{s^3} - \frac{6(s+5)^2}{s^4}\right]_{s=-1}$$

$$= \frac{1}{6}(-2 - 2 - 16 - 2 - 16 - 16 - 96) = -25$$

$$f(t) = [25 - (8/3)t^3e^{-t} - 12t^2e^{-t} - 25te^{-t} - 25e^{-t}]u(t)$$