FALL 2013 EENG382 EXAM #2 NAME KEY

Problem #1 (20 pts)

13.64 a) Use the convolution integral to find the output voltage of the circuit in Fig. P13.50(a) if the input voltage is the rectangular pulse shown in Fig. P13.64.

b) Sketch $v_o(t)$ versus t for the time interval $0 \le t \le 10$ ms.

Figure P13.64

$v(t) = 16V\,[u(t) - u(t-T_0)]$

$T_0 = 4ms$

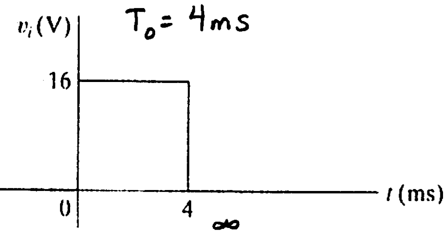
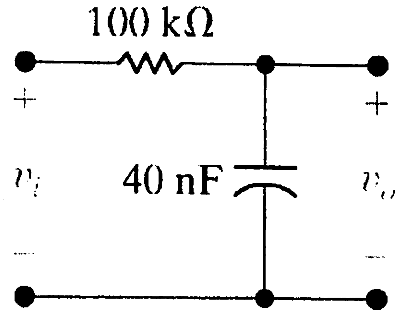


Figure P13.50



(a)

$$H(s) = \frac{Z_c}{R+Z_c} = \frac{\frac{1}{sC}}{R+\frac{1}{sC}} = \frac{1}{sRC+1}$$

$$H(s) = \frac{1}{RC}\cdot\frac{1}{s+\frac{1}{RC}} = \frac{1}{\tau}\cdot\frac{1}{s+1/\tau}$$

$$\tau = RC = (100k\Omega)(40nF) = 4\ ms$$

$$h(t) = \mathcal{L}^{-1}\{H(s)\} = \frac{1}{\tau}\mathcal{L}^{-1}\left\{\frac{1}{s+1/\tau}\right\}$$

$$\underline{h(t) = \frac{1}{\tau}e^{-\frac{t}{\tau}}u(t)}$$

$$v_o(t) = v_i(t) * h(t) = \int_{-\infty}^{\infty} v_i(\lambda)\,h(t-\lambda)\,d\lambda$$

$$h(t-\lambda) = \frac{1}{\tau}e^{-\frac{(t-\lambda)}{\tau}}u(t-\lambda) = \frac{1}{\tau}e^{-\frac{t}{\tau}}\cdot e^{\frac{\lambda}{\tau}}u(t-\lambda)$$

$$v_o(t) = \int_{-\infty}^{\infty} 16V[u(\lambda)-u(\lambda-T_0)]\,\frac{1}{\tau}e^{-\frac{t}{\tau}}e^{\frac{\lambda}{\tau}}u(t-\lambda)\,d\lambda$$

$$= \frac{16V}{\tau}e^{-\frac{t}{\tau}}\left[\int_{-\infty}^{\infty} e^{\frac{\lambda}{\tau}}\underbrace{u(\lambda)u(t-\lambda)}_{0\ \ldots\ t}d\lambda - \int_{-\infty}^{\infty} e^{\frac{\lambda}{\tau}}\underbrace{u(\lambda-T_0)u(t-\lambda)}_{T_0\ \ldots\ t}d\lambda\right]$$

$$= \frac{16V}{\tau}e^{-\frac{t}{\tau}}\left[\int_0^t e^{\frac{\lambda}{\tau}}d\lambda - \int_{T_0}^t e^{\frac{\lambda}{\tau}}d\lambda\right]$$

FOR $0 \le t \le T_0$

$$v_o(t) = \frac{16V}{\tau}e^{-\frac{t}{\tau}}\int_0^t e^{\frac{\lambda}{\tau}}d\lambda = \frac{16V}{\tau}e^{-\frac{t}{\tau}}\cdot\tau e^{\frac{\lambda}{\tau}}\Big|_0^t = 16V\,e^{-\frac{t}{\tau}}\left(e^{\frac{t}{\tau}}-1\right)$$

$$v_o(t) = 16V\left(1-e^{-\frac{t}{\tau}}\right) = 16V\left(1-e^{-\frac{t}{4ms}}\right)$$

FOR $T_0 \le t$

$$v_o(t) = \frac{16V}{\tau}e^{-\frac{t}{\tau}}\int_0^{T_0} e^{\frac{\lambda}{\tau}}d\lambda = \frac{16V}{\tau}e^{-\frac{t}{\tau}}\tau e^{\frac{\lambda}{\tau}}\Big|_0^{T_0} = 16V\,e^{-\frac{t}{\tau}}\left(e^{\frac{T_0}{\tau}}-1\right) = 16V$$

$$v_o(t) = 16V\left(e^{\frac{4ms}{4ms}}-1\right)e^{-\frac{t}{4ms}} = 16(e-1)V\,e^{-\frac{t}{4ms}} = 27.49V\,e^{-t/4ms}$$

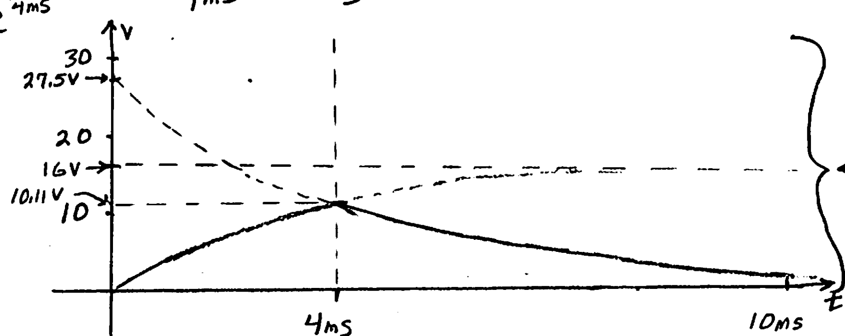
(#1a)

$$v_o(t) = \begin{cases} 0V & t<0 \\ 16V\left(1-e^{-\frac{t}{4ms}}\right) & 0\le t\le 4ms \\ 27.5V\,e^{-\frac{t}{4ms}} & 4ms < t \end{cases}$$

$v_o(4ms) = 16V\cdot(1-e^{-1})$
$= 10.11V$

$v_o(4ms) = 27.49V\,e^{-1}$
$= 10.11V$

AGREE ✓



(#1b)

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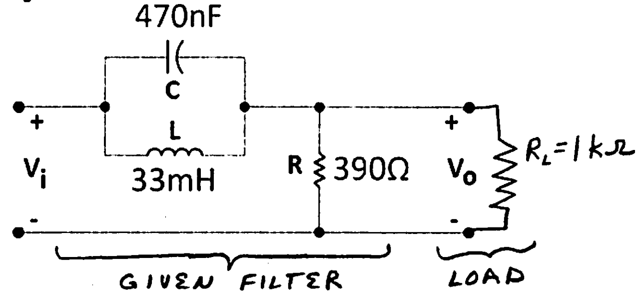
Problem #2 (20 pts)

Figure P14.36



14.37 Assume the bandreject filter in Problem 14.36 is loaded with a 1 kΩ resistor.

PSPICE MULTISIM

a) What is the quality factor of the loaded circuit?

b) What is the bandwidth (in kilohertz) of the loaded circuit?

c) What is the upper cutoff frequency in kilohertz?

d) What is the lower cutoff frequency in kilohertz?

$$R' = (R \| R_L) = \frac{(390\Omega)(1000\Omega)}{(1390\Omega)} = 280.6\Omega$$

$$H(s) = \frac{V_o}{V_i} = \frac{R'}{R' + (Z_L \| Z_C)} = \frac{R'}{R' + \frac{Z_L Z_C}{Z_L + Z_C}} = \frac{R'(Z_L + Z_C)}{R'(Z_L + Z_C) + Z_L Z_C}$$

$$= \frac{R'\left(sL + \frac{1}{sC}\right)}{R'\left(sL + \frac{1}{sC}\right) + (sL)\left(\frac{1}{sC}\right)} = \frac{\left(R'Ls + \frac{R'}{sC}\right)}{\left(R'Ls + \frac{L}{C} + \frac{1}{s}\cdot\frac{R'}{C}\right)} \cdot \frac{\left(\frac{s}{R'L}\right)}{\left(\frac{s}{R'L}\right)}$$

$$H(s) = \frac{\left(s^2 + \frac{1}{LC}\right)}{\left(s^2 + \frac{1}{R'C}s + \frac{1}{LC}\right)} \Rightarrow s^2 + \beta s + \omega_o^2$$

$$Q = \frac{\omega_o}{\beta} = \frac{\sqrt{\frac{1}{LC}}}{\frac{1}{R'C}} = \frac{R'C}{\sqrt{LC}} = \sqrt{\frac{R'^2C^2}{LC}} = R'\sqrt{\frac{C}{L}} = 280.6\Omega\sqrt{\frac{470nF}{}}$$

$$= 280.6\Omega\sqrt{\frac{470nF}{33mH}} = 280.6\Omega\sqrt{\frac{470n\,\frac{sec}{\Omega}}{33m\,\frac{\Omega}{sec}}} = 280.6\Omega\cdot\sqrt{\frac{470}{33}\mu\left(\frac{1}{\Omega^2}\right)}$$

(#2a)

$$\underline{\underline{Q = 1.059}} \leftarrow$$

$$\beta = \frac{1}{R'C} = \frac{1}{280.6\Omega \cdot 470n\frac{sec}{\Omega}} \cdot \frac{1\,Hz}{2\pi\,r/sec} = 1209.4\,Hz$$

(#2b)

$$\underline{\underline{\beta = 1.209\,kHz}} \leftarrow$$

$$f_{c1}f_{c2} = f_o^2;\; f_{c1} = f_{c2} - \beta = f_{c2} - \frac{f_o}{Q};\; \left(f_{c2} - \frac{f_o}{Q}\right)f_{c2} = f_o^2$$

$$f_{c2}^2 - \frac{f_o}{Q}f_{c2} - f_o^2 = 0$$

$$f_{c2} = \frac{1}{2}\left(\frac{f_o}{Q} \pm \sqrt{\left(\frac{f_o}{Q}\right)^2 + 4f_o^2}\right) = f_o\left(\frac{1}{2Q} \pm \sqrt{\left(\frac{1}{2Q}\right)^2 + 1}\right) = \frac{1}{\sqrt{LC}}\left(\frac{1}{2Q} \pm \sqrt{\left(\frac{1}{2Q}\right)^2 + 1}\right)$$

$$f_{c2} = \frac{1}{\sqrt{33mH \cdot 470nF}} \cdot \frac{1\,Hz}{2\pi\,r/s} \cdot \left(\frac{1}{2\cdot 1.059} \oplus \sqrt{\left(\frac{1}{2\cdot 1.059}\right)^2 + 1}\right) = \underbrace{1.2780\,kHz}_{f_o} \cdot 1.5780$$

(#2c)

$$\underline{\underline{f_{c2} = 2.02\,kHz}} \leftarrow$$

$$f_{c1} = f_{c2} - \beta = 2.02kHz - 1.209kHz = 0.807\,kHz$$

(#2d)

$$\underline{\underline{f_{c1} = 0.807\,kHz}} \leftarrow$$

CHECK: $f_o \stackrel{?}{=} \sqrt{f_{c1}f_{c2}} = \sqrt{2.02kHz \cdot 0.807kHz} = 1.277\,kHz$ ✓

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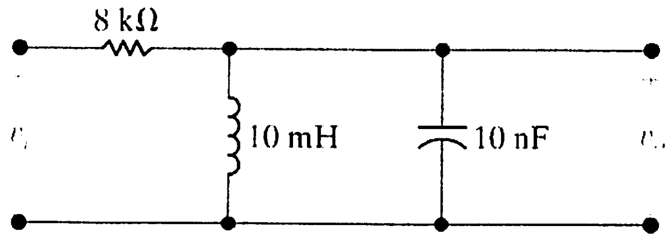
Problem #3 (20 pts)

15.24 Scale the bandpass filter in Problem 14.22 so that the center frequency is 200 kHz and the quality factor is still 8, using a 2.5 nF capacitor. Determine the values of the resistor, the inductor, and the two cutoff frequencies of the scaled filter.

Figure P14.22



ORIGINAL FILTER: $\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10mH\cdot 10nF}} = \frac{1}{\sqrt{100\,p\,(s^2)}} = \frac{1}{10\mu s} = 100\ kr/s$

$f_o = 100\ kr/s \cdot \frac{1\,cyc}{2\pi\,rad}\cdot\frac{1\,Hz}{1\,cyc/s} = 15.916\ kHz$

$\beta = \frac{f_o}{Q} = \frac{15.916\,kHz}{8} = 1.9894\ kHz$

$f_o = \sqrt{f_{c1}f_{c2}}\ ;\ f_{c2} - f_{c1} = \beta = \frac{f_o}{Q}$

$f_o^2 = f_{c1}f_{c2} = f_{c1}\left(f_{c1} + \frac{f_o}{Q}\right)$

$f_{c1}^2 + \frac{f_o}{Q}f_{c1} - f_o^2 = 0$

$f_{c1} = \left(-\frac{f_o}{Q} \pm \sqrt{\left(\frac{f_o}{Q}\right)^2 + 4f_o^2}\right)\frac{1}{2} = -\frac{f_o}{2Q} \pm f_o\sqrt{\frac{1}{4Q^2}+1}$

$f_{c1} = f_o\left(\sqrt{\frac{1}{4Q^2}+1} - \frac{1}{2Q}\right) = 15.916\,kH\cdot\left(\sqrt{\frac{1}{4\cdot 32}+1} - \frac{1}{2\cdot 8}\right) = 15.916\,kHz\cdot \underline{94.14\%}$

SCALING FACTORS: $k_f = \frac{f_o'}{f_o} = \frac{200\,kHz}{15.916\,kHz} = \underline{12.566}$

$C' = \frac{1}{k_f k_m}C \Rightarrow k_m = \frac{1}{k_f}\cdot\frac{C}{C'} = \frac{1}{12.566}\cdot\frac{10nF}{2.5nF} = \underline{0.3183}$

SCALED COMPONENTS: $R' = k_m R = 0.3183 \cdot 8\,k\Omega = \underline{2.546\,k\Omega}$

$L' = \frac{k_m}{k_f}L = \frac{0.3183}{12.566}\cdot 10\,mH = \underline{253.3\,\mu H}$

NEW FILTER: $\omega_o' = \frac{1}{\sqrt{L'C'}} = \frac{1}{\sqrt{0.2533mH\cdot 2.5nF}}\cdot\frac{1\,Hz}{2\pi\,r/s} = 200\,kHz$ ✓

$f_{c1}' = f_o\cdot 94.14\% = 200\,kHz\cdot 94.14\% = \underline{188.28\,kHz}$

$f_{c2}' = f_{c1}' + \beta' = f_{c1}' + \frac{f_o}{Q} = 188.28\,kHz + \frac{200\,kH}{8} = \underline{\underline{213.28\,kHz}}$

ANSWERS:

$$\left.\begin{aligned} R' &= 2.55\,k\Omega \\ L' &= 253\,\mu H \\ f_{c1}' &= 188.3\,kHz \\ f_{c2}' &= 213\,kHz \end{aligned}\right\}$$

(#3)