On Multiple Access Interference in a DS/FFH Spread Spectrum Communication system

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Abstract

This paper describes the influence of Multiple Access (MA) interference on the performance of a non-coherent hybrid Direct Sequence Fast Frequency Hopping SSMA Communication system applying FSK-modulation. The MA-interference plays an important role in determining the total interference in a Spread Spectrum system, especially in a non-cellular environment, where Power Control is hardly possible. A cross correlation factor which is directly responsible for this interference is derived. Using this factor, a comparison between slow and fast frequency hopping (both in combination with direct sequence) is made. At the end two sets of Kasami codes are selected which offer a good performance.

1 Introduction

This paper deals with a hybrid Direct Sequence, Fast Frequency Hop system in a non-cellular environment. Whereas most publications so far ([1, 2] and others) put emphasis on long hopping-patterns, we focus on short sequences resulting in less bandwidth occupancy. We also assume a spatial distribution of users which results in a non-equal power reception for different users.

A non-cellular environment leads to a point-to-point communication system without a base station. This is a more flexible and less expensive method than the cellular approach, but inhibits power control, playing a key role in reducing the Near-Far effect [3, 4]. The hybrid DS/FFH technique is applied to both beat the Near-Far effect and to retain the advantages of Direct Sequence: Jamming rejection, fading rejection and Security.

In section 2 the hybrid system-model is described. Section 3 deals with the MA-interference, here the measure for MA-interference in a DS/FFH system is derived, and the relation with the S/I-ratio is mentioned. In section 4 a comparison is made between DS/SFH and DS/FFH for Rayleigh fading channels. Section 5 provides two sets of Kasami-codes which offer good MA-interference properties. Finally section 6 gives some conclusions.

2 System Model

2.1 DS/FFH Spread Spectrum technique

The DS/FFH Spread Spectrum technique combines the advantages of the direct-sequence and frequency-hopping spreading techniques while compensating for the others. Each data bit is divided over a number (N_{FH}) of frequency-hop channels (carrier frequencies). In each frequency-hop channel a complete PN-sequence of length N is combined with the data signal (see figure 1 where $N_{FH} = 5$). Applying Fast Frequency Hopping (FFH) requires a wider bandwidth than Slow Frequency Hopping (SFH). This increase however, is marginal compared to the enormous bandwidth already in use.

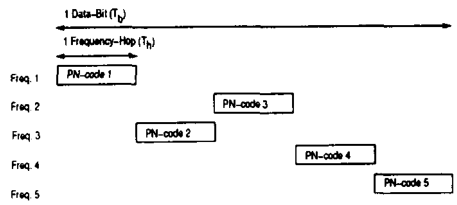

Figure 1: DS/FFH Spreading scheme

Since the FH-sequence and the PN-codes are coupled, every receiver is identified by a combination of an FH-sequence and N_{FH} PN-codes. To limit the probability that two users share the same frequency channel simultaneously, frequency-hop sequences are chosen in such a way that two transmitters with different FH-sequences share at most two frequencies at the same time in one bit-period (asynchronous transmission).

Applying several frequency hops within each data bit disqualifies modulation by some kind of Phase Shift Keying. PSK is also quite susceptible to channel distortions. An FSK modulation scheme is therefore chosen.

2.2 The DS/FFH SSMA system model

One of the important advantages of spread spectrum systems is their multiple access capability. The interference that results from simultaneous transmittance is called 'Multiple Access (MA) Interference'. Contribution to this interference occurs when the reference user and a non-reference user use the same FH-channel for a fraction of a frequency-hop.

In case of pure DS all codes are transmitted in the same frequency slot, so the codes will correlate completely. In the DS/FFH case, two subsequent codes are transmitted in different frequency slots, so those codes do correlate only partially. In figure 2 this is shown in more detail.

The DS/FFH SSMA system model that will be considered is shown in figure 3.

Suppose there are K active users, one of them, user i, being the reference user. The other $K-1$ users cause MA-interference.

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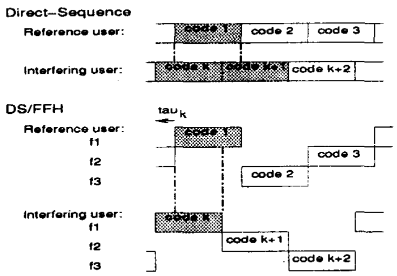

Figure 2: DS versus DS/FFH

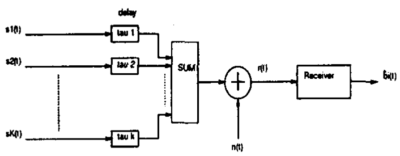

Figure 3: DS/FFH system model

The data signal, $b_k(t)$, is a polar bit stream of unit amplitude, with a duration T_b. This signal is FSK-modulated. The FSK-spacing is Δ_{FSK}. $a_k(t)$ is the waveform of the PN-code, also a sequence of polar bits with unit amplitude, with duration T_c. If we denote by $d_k(t)$ the sequence $\{-\frac{N_{FH}}{2}+\frac{1}{2}, -\frac{N_{FH}}{2}+1\frac{1}{2}, \cdots, \frac{N_{FH}}{2}-\frac{1}{2}\}$ of duration $T_h = NT_c = T_b/N_{FH}$, the transmitted signal of the k-th user is:

$$\begin{aligned} s_k(t) &= \sqrt{2P}sin\{(\omega_c + d_k(t)\Delta_{FH} \\ &\quad +\frac{1}{2}b_k(t)\Delta_{FSK})t + \frac{\pi}{2}a_k(t) + \theta_k\} \\ &= \sqrt{2P}a_k(t)cos\{(\omega_c + d_k(t)\Delta_{FH} \\ &\quad +\frac{1}{2}b_k(t)\Delta_{FSK})t + \theta_k\} \end{aligned} \tag{1}$$

Δ_{FH} is the frequency-hop spacing. P is the common transmitted power, ω_c the center-frequency and θ_k the phase of the k-th carrier.

For the time being we assume that collision between the k-th and the i-th user in one frequency-hop channel always occurs. Later this can be corrected by evaluating the hit chance [5]. The FH-spacing is chosen so that double frequency terms can be removed after modulation ($\Delta_{FH} \gg T_h^{-1}$).

The received signal is now:

$$\begin{aligned} r(t) = &\sum_{k=1}^{K} \sqrt{2P}a_k(t)cos\{(\omega_c + \\ &d_k(t)\Delta_{FH} + \tfrac{1}{2}b_k(t)\Delta_{FSK})t\} + n(t) \end{aligned} \tag{2}$$

A correlation receiver, synchronized with user i, responds to $r(t)$ with an output

$$\begin{aligned} Z_i = &\sqrt{2P}\{T_h + \sum_{k=1,k\neq i}^{K} R_{k,i}(\tau_k)\cdot cos\phi_k\} \\ &+\int_0^{T_h} n(t)a_i(t)cos(\cdots)dt \end{aligned} \tag{3}$$

Here $\phi_k = \theta_k + \omega_c\tau_k$, τ_k is the delay of the k-th user (τ_k is defined to be positive if user i is delayed with regard to user k). So τ_k can vary between $-T_h$ and T_h, $n(t)$ is the channel noise, $cos(\cdots)$ denotes the same cos-term as in (2) and $R_{k,i}(\tau_k)$ is a continuous time partial cross correlation function defined by:

$$R_{k,i}(\tau_k) = \int_{\tau_{k1}}^{\tau_{k2}} a_k(t+\tau_k)a_i(t)dt \tag{4}$$

In the DS-case there are two correlation terms [6]. The interval (τ_{k1}, τ_{k2}) is the time that two users i and k share the same frequency-hop channel. τ_{k1} and τ_{k2} are related to τ_k in the following way (see also figure 2):

$$\tau_{k1} = \begin{cases} 0, & \tau_k \leq 0 \\ \tau_k, & \tau_k > 0 \end{cases}$$

$$\tau_{k2} = \begin{cases} \tau_k + T_h, & \tau_k \leq 0 \\ T_h, & \tau_k > 0 \end{cases}$$

If $-T_h \leq lT_c \leq \tau_k \leq (l+1)T_c \leq T_h$,l being an integer, this correlation function can be written as:

$$\begin{aligned} R_{k,i}(\tau_k) = &C_{k,i}(l)\{(l+1)T_c - \tau_k\} \\ &+C_{k,i}(l+1)(\tau_k - lT_C) \end{aligned} \tag{5}$$

$C_{k,i}(l)$ is the discrete aperiodic cross correlation function [7]:

$$C_{k,i}(l) = \begin{cases} \sum_{j=0}^{N-1-l} a_j^{(k)}a_{j+l}^{(i)}, & 0 \leq l \leq N-1 \\ \sum_{j=0}^{N-1+l} a_{j-l}^{(k)}a_j^{(i)}, & 1-N \leq l < 0 \\ 0, & |l| \geq N \end{cases}$$

3 MA-interference parameter

In this section an expression for the measure of the MA-interference is derived. We use results from [6] and [8] extended to a DS/FFH SSMA system.

In this approach phase shifts and time delays are treated as independent random variables. As we are only interested in choosing a good set of PN-codes, the only interference considered is MA-interference. ϕ_i and τ_i are assumed to be 0.

The desired signal component of Z_i is now $\sqrt{P/2}T_h$ while the variance of the MA-component of Z_i is (under Gaussian assumption):

$$Var\{Z_{i,MA}\} = \frac{P}{8T_h}\sum_{k=1,k\neq i}^{K}\int_{-T_h}^{T_h} R_{k,i}^2(\tau)d\tau$$
$$= \frac{P}{8T_h}\sum_{k=1,k\neq i}^{K}\sum_{l=1-N}^{N-1}\int_{lT_c}^{(l+1)T_c} R_{k,i}^2(\tau)d\tau + \frac{N_0T_h}{4} \quad (6)$$

As τ can take values between $-T_c$ and T_c, τ is integrated over the corresponding interval. Substituting $R_{k,i}(\tau)$ from (5) and evaluating the integral gives:

$$Var\{Z_{i,MA}\} = \frac{PT_h^2}{24N^3}\left\{\sum_{k=1,k\neq i}^{K}\rho_{k,i}\right\} + \frac{N_0T_h}{4} \quad (7)$$

where:

$$\rho_{k,i} = \sum_{l=1-N}^{N-1}\{C_{k,i}^2(l) + C_{k,i}(l)C_{k,i}(l+1) + C_{k,i}^2(l+1)\}$$

After some calculation we find for the mean and variance of $\rho_{k,i}$:

$$E[\rho_{k,i}] = 2N^2 - 1 \quad (8)$$

$$Var[\rho_{k,i}] = 6N^3 - 5N^2 + 2N \quad (9)$$

Interesting to mention is the relation between this factor and the $r_{k,i}$-factor from [6]: $\rho_{k,i} = r_{k,i} - 1$. Since the $\rho_{k,i}$-term is directly responsible for the MA-interference in a DS/FFH system, proper PN-codes can be selected on the basis of this term.

Assuming the mean value for $\rho_{k,i}$, the resulting signal to interference ratio (S/I) becomes:

$$S/I_{FFH} = \frac{2E_{b,i}/N_0}{1 + \frac{1}{6N}\cdot\frac{2}{N_0}\sum_{k=0,k\neq i}^{K}E_{b,k}} \quad (10)$$

Here $E_{b,k}$ is the received energy per bit for the k^{th} transmitter, N_0 is the single-sided spectral noise density, and K is the number of users which share together with the reference user the same frequency hop channel at the same time (they have a hit).

In this formula we did not account for the diversity which is inherent to fast frequency hopping. As only one bit is transmitted in N_{FH} frequency slots, it is possible to detect all frequency hops separately and then make a majority vote, applying this strategy yields an enhanced performance.

When applying majority voting the signal to interference ratio (per frequency hop) changes:

$$S/I_{FFH,MV} = \frac{2E_{h,i}/N_0}{1 + \frac{1}{6N}\cdot\frac{2}{N_0}\sum_{k=0,k\neq i}^{K}E_{b,k}} \quad (11)$$

Here E_h is the energy per frequency hop.

One last remark is to be made. In the derivation of the signal to interference ratio the signals are assumed to be Gaussian, this assumption is only valid for either long PN-codes or a large number of users. Simulations show that for a FH-pattern with maximal two hits and a PN-code length of 63, 2 interfering users (equal power) justify this assumption.

4 FFH versus SFH

This section provides a comparison the performance of DS/FFH and DS/SFH systems, on the basis of the same bandwidth occupancy. As this section is only meant as an illustration, we will not go into detail but give references instead.

For $\rho_{k,i}$ a value of $2N^2$ is chosen ($E[\rho_{k,i}] = 2N^2 - 1$), in section 5 will be shown that sets of PN-codes can be found which have $2N^2$ as an upper bound. For N_{FH} we chose a value of 7. Now there are 6 sequences possible with a length of 7 which share maximally two frequencies simultaneously during 1 hop-time (random time shift). For the DS-spreading the PN-code length is 63.

The chance that two transmitters share the same frequency slot simultaneously (P_{hit}) is evaluated in [1] and is equal to $1/q$ in the FFH-case, q being the number of frequency-hop channels. For the SFH case we find the following formula:

$$P_{hit,\,SFH} = \frac{1 - N_b^{-1}}{q} + \frac{2}{N_bq} \quad (12)$$

with N_b equal to the number of bits per hop.

For the S/I-ratio the formula by Pursley will be used [6] (The $r_{k,i}$-factor will also be approximated by its mean: $2N^2$):

$$S/I_{SFH} = \frac{2E_{b,i}/N_0}{1 + \frac{1}{3N}\cdot\frac{2}{N_0}\sum_{k=0,k\neq i}^{K}E_{b,k}} \quad (13)$$

As no power control is assumed an assumption towards the spatial distribution of the users is to be made. For this we will use the log-normal distribution function motivated in [9]:

$$K'(r) = \frac{K-1}{\sqrt{2\pi}rs_i}exp\left[-\frac{(lnr)^2}{2s_i^2}\right] \quad (14)$$

Here $K'(r)$ is the number of interfering users per unit distance at a distance r. s_i is a parameter which determines the shape of the distribution, this parameter is chosen to have a value of 0.7.

The power is expected to decrease as a function of the normalized distance: $P(r) \sim r^{-\beta}$, for β a typical value of 3 has been chosen. The reference user is assumed to have a normalized distance

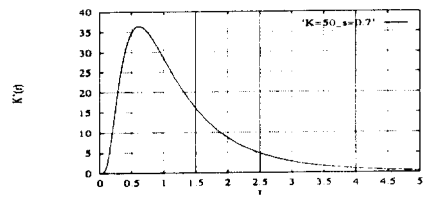

Figure 4: Spatial distribution of users

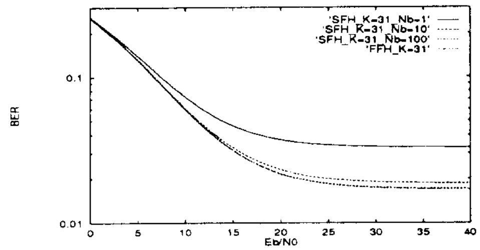

Figure 6: Influence of N_b for Slow Frequency Hopping

$r = 0.7$ to the receiver. The resulting spatial distribution for 50 users is shown in figure 4.

The channel is assumed to have Rayleigh fading, so the relation between the S/I-ratio (γ) and the Bit Error Rate for the FSK modulation scheme is given by [10]:

$$BER = \frac{1}{2+\gamma} \qquad (15)$$

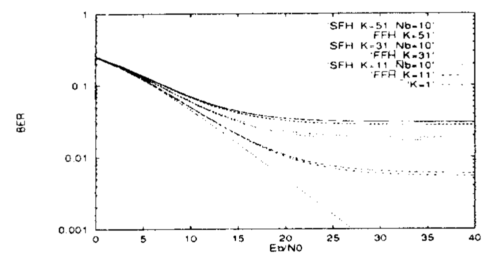

Figure 5: BER as a function of Eb/N_0 for both SFH and FFH, no diversity

When comparing DS/FFH and DS/SFH without applying the diversity inherent to FFH both systems will perform about the same. There is only an advantage of using FFH over SFH if N_b, the number of bits per frequency hop, is limited. A limitation on this point can be introduced by an applied error correcting code (and so by the amount of memory in the receiver). In figure 5 the BER versus E_b/N_0 curves are drawn for SFH ($N_b = 10$) and FFH, in this example q and N_{FH} have a value of 7. The number of active users (K) is varied between 1 and 51. Figure 6 shows the influence of changing the number of bits per hop for DS/SFH ($K = 31$, $q = N_b = 7$).

More interesting is the case in which we apply diversity (majority voting). Results are given in figure 7 for 1, 11, 31 and 51 users, q and N_{FH} have a value of 7. For majority voting it is assumed that the interference of two successive 'hops' is independent (different frequency channels, pseudo-random hop-sequences), this is not true in the DS/SFH case.

5 Finding a proper set of PN-codes

In section 3 a measure was derived for the MA-interference in a DS/FFH SSC system. This measure will function as a criterion

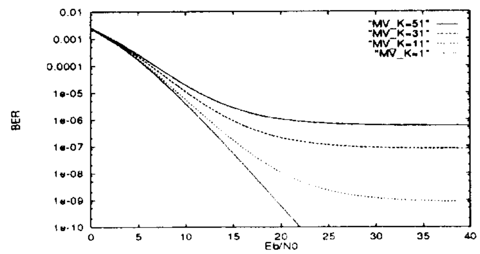

Figure 7: BER as a function of the S/N-ratio for FFH exploiting diversity

to select a proper set of PN-codes. Except for a low cross-correlation factor the codes also have to meet the "balance"-property: the difference between the number of ones and zeros in the code may only be 1. This last requirement stands for good spectral density properties. Candidates are codes from the large set of Kasami codes, the code-length is chosen to be 63. In [11] also code-set selections can be found, candidates here are the Gold codes and the small set of Kasami codes, the "balance"-criterion is not met in that case.

Kasami codes in the large set are created by combination of 3 m-sequences [8, 12]:

$$r = u \cdot T^k v \cdot T^m w$$

Here u and v are m-sequences of length: $N = 2^n - 1$ (n even) which form a preferred pair [13]. w is a m-sequence resulting after decimation the v-code with a value $2^{n/2} + 1$. T denotes a delay of one element, k is the offset of the v-code with respect to the u-code and m is the offset of the w-code with respect to the u-code. Offsets are relative to the all-ones state.

The notation from (16) will be used for Kasami-codes of the large set with $n = 6$ ($N = 63$), u is the m-sequence with feedbacks (6,1) and v is the m-sequence with feedbacks (6,5,2,1).

To calculate the total MA-interference in a SSC system, we have to know the $\rho_{k,i}$-factor of each code combination used. The mean-value of this term is $2N^2 - 1$, as a selection-criterion we require that for all code combinations in the set the $\rho_{k,i}$-factor has a value less than this mean ($\rho_{k,i} < 2N^2 - 1$).

$$
kas(n,k,m) = \begin{cases} u \cdot T^k v \cdot T^m w, & \\ \quad 0 \le k \le 2^n - 2, 0 \le m \le 2^{n/2} - 2 & \\ u \cdot T^m w, \ (small\ set) & \\ \quad k = 2^n - 1, 0 \le m \le 2^{n/2} - 2 & \\ v \cdot T^m w, & \\ \quad k = 2^n, 0 \le m \le 2^{n/2} - 2 & \\ u \cdot T^k v, \ (Gold\ codes) & \\ \quad 0 \le k \le 2^n - 2, m = 2^{n/2} - 1 & \\ v, \ (m - sequence) & \\ \quad k = 2^n - 1, m = 2^{n/2} - 1 & \\ u, \ (m - sequence) & \\ \quad k = 2^n, m = 2^{n/2} - 1 & \end{cases} \tag{16}
$$

Let us look at a set of PN-codes with $\rho_{k,i}$-factors below $2N^2 - 1$. The PN-codes in this example have length 63, so there are 520 possible PN-codes [8]. It appeared that among these 520 codes only 241 codes are balanced, these 241 balanced codes are taken as a candidate set.

It turned out that there are two sets, each containing 17 PN-codes, for which all $\rho_{k,i}$-factors have values below $2N^2$. These sets are shown in the tables 1 and 2.

Table 1: set of PN-codes with $\rho_{k,i} < 2N^2 - 1$ with $N = 63$

kas(6,46,3)	kas(6,50,0)	kas(6,57,0)
kas(6,60,0)	kas(6,14,1)	kas(6,21,1)
kas(6,26,1)	kas(6,44,1)	kas(6,51,1)
kas(6,8,3)	kas(6,16,3)	kas(6,1,4)
kas(6,17,4)	kas(6,30,4)	kas(6,3,5)
kas(6,31,5)	kas(6,64,7)	

Table 2: set of PN-codes with $\rho_{k,i} < 2N^2 - 1$ with $N = 63$

kas(6,50,0)	kas(6,60,0)	kas(6,26,1)
kas(6,44,1)	kas(6,51,1)	kas(6,53,3)
kas(6,38,4)	kas(6,44,4)	kas(6,3,5)
kas(6,11,5)	kas(6,31,5)	kas(6,32,5)
kas(6,41,5)	kas(6,62,5)	kas(6,0,6)
kas(6,18,6)	kas(6,64,7)	

To determine the number of addresses which can be formed with these sets of codes, recall that if there are $(N_{FH} + 1)$ FH-channels available, N_{FH} FH-sequences can be formed. Another constraint is that one user address uses N_{FH} different PN-codes (to avoid the case that two addresses have the frequency-hop sequence as well as more than one PN-code in common). When $N_c = 17$ (number of codes in a code set) the resulting number of addresses is then about $8 \cdot 10^6$.

6 Conclusions

To be able to reduce the MA-interference in a DS/FFH SSMA communication system a criterion for choosing sets of PN-codes was provided. First a factor ($\rho_{k,i}$) is derived which is directly responsible for the MA-interference in DS/FFH SSMA systems. Comparing this factor with a corresponding factor in a DS/SFH or DS SSC system [6] it appears that, if no diversity is applied, DS/FFH systems only outperforms DS/SFH if the number of bits per hop in the latter case is limited. However when exploiting the diversity inherent to Fast Frequency Hopping, the DS/FFH system performs much better. This is illustrated by BER versus E_b/N_0 plots for both techniques.

In most applications a lot of users have to be allowed in the system. In such a case the Bit Error Rate is almost totally determined by the MA-interference. This makes selecting sets of PN-codes with good cross-correlation properties necessary. The sets presented in section 5 are the proper sets to use under these circumstances.

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