

EENG382 QZ03 – AUTHOR'S SOLUTIONS

NOTE: I have not yet verified that the author's solutions are, in fact, correct.

Prob 16.22

P 16.22 [a] The current has half-wave symmetry. Therefore,

$$a_v = 0; \quad a_k = b_k = 0, \quad k \text{ even}$$

For k odd,

$$a_k = \frac{4}{T} \int_0^{T/2} \left(I_m - \frac{2I_m}{T}t \right) \cos k\omega_0 t \, dt \quad T/2$$

$$= \frac{4}{T} \int_0^{T/2} I_m \cos k\omega_0 t \, dt - \frac{8I_m}{T^2} \int_0^{T/2} t \cos k\omega_0 t \, dt \quad 0$$

$$= \frac{4I_m}{T} \frac{\sin k\omega_0 t}{k\omega_0} \Big|_0^{T/2} - \frac{8I_m}{T^2} \left[\frac{\cos k\omega_0 t}{k^2\omega_0^2} + \frac{t}{k\omega_0} \sin k\omega_0 T \right]_0^{T/2}$$

$$= 0 - \frac{8I_m}{T^2} \left[\frac{\cos k\pi}{k^2\omega_0^2} - \frac{1}{k^2\omega_0^2} \right]$$

$$= \left(\frac{8I_m}{T^2} \right) \left(\frac{1}{k^2\omega_0^2} \right) (1 - \cos k\pi)$$

$$= \frac{4I_m}{\pi^2 k^2} = \frac{20}{k^2}, \quad \text{for } k \text{ odd}$$

$$b_k = \frac{4}{T} \int_0^{T/2} \left(I_m - \frac{2I_m}{T}t \right) \sin k\omega_0 t \, dt$$

$$= \frac{4I_m}{T} \int_0^{T/2} \sin k\omega_0 t \, dt - \frac{8I_m}{T^2} \int_0^{T/2} t \sin k\omega_0 t \, dt$$

$$= \frac{4I_m}{T} \left[\frac{-\cos k\omega_0 t}{k\omega_0} \right]_0^{T/2} - \frac{8I_m}{T^2} \left[\frac{\sin k\omega_0 t}{k^2\omega_0^2} - \frac{t}{k\omega_0} \cos k\omega_0 T \right]$$

$$= \frac{4I_m}{T} \left[\frac{1 - \cos k\pi}{k\omega_0} \right] - \frac{8I_m}{T^2} \left[\frac{-T \cos k\pi}{2k\omega_0} \right]$$

$$= \frac{8I_m}{k\omega_0 T} \left[1 + \frac{1}{2} \cos k\pi \right]$$

$$= \frac{2I_m}{\pi k} = \frac{10\pi}{k}, \quad \text{for } k \text{ odd}$$

Prob 16.22 (Cont'd)

$$a_k - jb_k = \frac{20}{k^2} - j\frac{10\pi}{k} = \frac{10}{k} \left(\frac{2}{k} - j\pi \right) = \frac{10}{k^2} \sqrt{\pi^2 k^2 + 4} \angle -\theta_k$$

$$\text{where } \tan \theta_k = \frac{\pi k}{2}$$

$$i(t) = 10 \sum_{n=1,3,5,\dots}^{\infty} \frac{\sqrt{(n\pi)^2 + 4}}{n^2} \cos(n\omega_0 t - \theta_n), \quad \theta_n = \tan^{-1} \frac{n\pi}{2}$$

$$[\mathbf{b}] \quad A_1 = 10\sqrt{4 + \pi^2} \cong 37.24 \text{ A} \quad \tan \theta_1 = \frac{\pi}{2} \quad \theta_1 \cong 57.52^\circ$$

$$A_3 = \frac{10}{9} \sqrt{4 + 9\pi^2} \cong 10.71 \text{ A} \quad \tan \theta_3 = \frac{3\pi}{2} \quad \theta_3 \cong 78.02^\circ$$

$$A_5 = \frac{10}{25} \sqrt{4 + 25\pi^2} \cong 6.33 \text{ A} \quad \tan \theta_5 = \frac{5\pi}{2} \quad \theta_5 \cong 82.74^\circ$$

$$A_7 = \frac{10}{49} \sqrt{4 + 49\pi^2} \cong 4.51 \text{ A} \quad \tan \theta_7 = \frac{7\pi}{2} \quad \theta_7 \cong 84.80^\circ$$

$$A_9 = \frac{10}{81} \sqrt{4 + 81\pi^2} \cong 3.50 \text{ A} \quad \tan \theta_9 = \frac{9\pi}{2} \quad \theta_9 \cong 85.95^\circ$$

$$\begin{aligned} i(t) &\cong 37.24 \cos(\omega_0 t - 57.52^\circ) + 10.71 \cos(3\omega_0 t - 78.02^\circ) \\ &\quad + 6.33 \cos(5\omega_0 t - 82.74^\circ) + 4.51 \cos(7\omega_0 t - 84.80^\circ) \\ &\quad + 3.50 \cos(9\omega_0 t - 85.95^\circ) + \dots \end{aligned}$$

$$\begin{aligned} i(T/4) &\cong 37.24 \cos(90 - 57.52^\circ) + 10.71 \cos(270 - 78.02^\circ) \\ &\quad + 6.33 \cos(450 - 82.74^\circ) + 4.51 \cos(630 - 84.80^\circ) \\ &\quad + 3.50 \cos(810 - 85.95^\circ) \cong 26.23 \text{ A} \end{aligned}$$

Actual value:

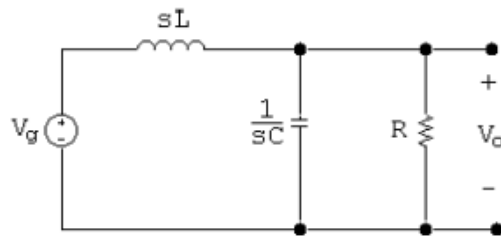
$$i\left(\frac{T}{4}\right) = \frac{1}{2}(5\pi^2) \cong 24.67 \text{ A}$$

Prob 16.35

$$P\ 16.35\ v_g = 10 - \frac{80}{\pi^2} \sum_{n=1,3,5,\dots}^{\infty} \frac{1}{n^2} \cos n\omega_o t\ V$$

$$\omega_o = \frac{2\pi}{T} = \frac{2\pi}{4\pi} \times 10^3 = 500\ \text{rad/s}$$

$$v_g = 10 - \frac{80}{\pi^2} \cos 500t - \frac{80}{9\pi^2} \cos 1500t + \dots$$



$$\frac{V_o - V_g}{sL} + sCV_o + \frac{V_o}{R} = 0$$

$$V_o(RLCs^2 + Ls + R) = RV_g$$

$$H(s) = \frac{V_o}{V_g} = \frac{1/LC}{s^2 + s/RC + 1/LC}$$

$$\frac{1}{LC} = \frac{10^6}{(0.1)(10)} = 10^6$$

$$\frac{1}{RC} = \frac{10^6}{(50\sqrt{2})(10)} = 1000\sqrt{2}$$

$$H(s) = \frac{10^6}{s^2 + 1000\sqrt{2}s + 10^6}$$

Prob 16.35 (Cont'd)

$$H(j\omega) = \frac{10^6}{10^6 - \omega^2 + j1000\omega\sqrt{2}}$$

$$H(j0) = 1$$

$$H(j500) = 0.9701 / \underline{-43.31^\circ}$$

$$H(j1500) = 0.4061 / \underline{-120.51^\circ}$$

$$v_o = 10(1) + \frac{80}{\pi^2}(0.9701) \cos(500t - 43.31^\circ)$$

$$+ \frac{80}{9\pi^2}(0.4061) \cos(1500t - 120.51^\circ) + \dots$$

$$v_o = 10 + 7.86 \cos(500t - 43.31^\circ) + 0.3658 \cos(1500t - 120.51^\circ) + \dots$$

$$V_{\text{rms}} \cong \sqrt{10^2 + \left(\frac{7.86}{\sqrt{2}}\right)^2 + \left(\frac{0.3658}{\sqrt{2}}\right)^2} = 11.44 \text{ V}$$

$$P \cong \frac{V_{\text{rms}}^2}{50\sqrt{2}} = 1.85 \text{ W}$$

Note – the higher harmonics are severely attenuated and can be ignored. For example, the 5th harmonic component of v_o is

$$v_{o5} = (0.1580) \left(\frac{80}{25\pi^2}\right) \cos(2500t - 146.04^\circ) = 0.0512 \cos(2500t - 146.04^\circ) \text{ V}$$

Prob 16.47

$$\text{P 16.47 [a]} \quad C_o = a_v = \frac{(1/2)(T/2)V_m}{T} = \frac{V_m}{4}$$

$$\begin{aligned} C_n &= \frac{1}{T} \int_0^{T/2} \frac{2V_m}{T} t e^{-jn\omega_o t} dt \\ &= \frac{2V_m}{T^2} \left[\frac{e^{-jn\omega_o t}}{-n^2\omega_o^2} (-jn\omega_o t - 1) \right]_0^{T/2} \\ &= \frac{V_m}{2n^2\pi^2} [e^{-jn\pi} (-jn\pi + 1) - 1] \end{aligned}$$

Since $e^{-jn\pi} = \cos n\pi$ we can write

$$C_n = \frac{V_m}{2\pi^2 n^2} (\cos n\pi - 1) + j \frac{V_m}{2n\pi} \cos n\pi$$

$$\text{[b]} \quad C_o = \frac{54}{4} = 13.5 \text{ V}$$

$$C_{-1} = \frac{-54}{\pi^2} + j \frac{27}{\pi} = 10.19/122.48^\circ \text{ V}$$

$$C_1 = 10.19/ -122.48^\circ \text{ V}$$

$$C_{-2} = -j \frac{13.5}{\pi} = 4.30/ -90^\circ \text{ V}$$

$$C_2 = 4.30/90^\circ \text{ V}$$

$$C_{-3} = \frac{-6}{\pi^2} + j \frac{9}{\pi} = 2.93/101.98^\circ \text{ V}$$

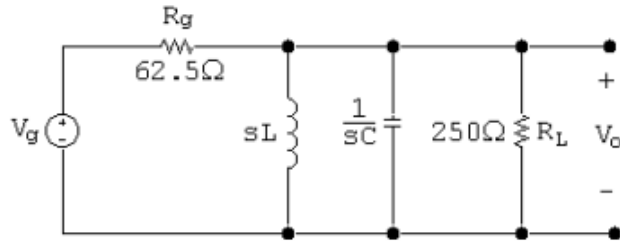
$$C_3 = 2.93/ -101.98^\circ \text{ V}$$

$$C_{-4} = -j \frac{6.75}{\pi} = 2.15/ -90^\circ \text{ V}$$

$$C_4 = 2.15/90^\circ \text{ V}$$

Prob 16.47 (Cont'd)

[c]



$$\frac{V_o}{250} + \frac{V_o}{sL} + V_o sC + \frac{V_o - V_g}{62.5} = 0$$

$$\therefore (250LCs^2 + 5sL + 250)V_o = 4sLV_g$$

$$\frac{V_o}{V_g} = H(s) = \frac{(4/250C)s}{s^2 + 1/50C + 1/LC}$$

$$H(s) = \frac{16,000s}{s^2 + 2 \times 10^4 s + 4 \times 10^{10}}$$

$$\omega_o = \frac{2\pi}{T} = \frac{2\pi}{10\pi} \times 10^6 = 2 \times 10^5 \text{ rad/s}$$

$$H(j0) = 0$$

$$H(j2 \times 10^5 k) = \frac{j8k}{100(1 - k^2) + j10k}$$

Therefore,

$$H_{-1} = 0.8/\underline{0^\circ}; \quad H_1 = 0.8/\underline{0^\circ}$$

$$H_{-2} = \frac{-j16}{-300 - j20} = 0.0532/\underline{86.19^\circ}; \quad H_2 = 0.0532/\underline{-86.19^\circ}$$

$$H_{-3} = \frac{-j24}{-800 - j30} = 0.0300/\underline{87.85^\circ}; \quad H_2 = 0.0300/\underline{-87.85^\circ}$$

$$H_{-4} = \frac{-j32}{-1500 - j40} = 0.0213/\underline{88.47^\circ}; \quad H_2 = 0.0213/\underline{-88.47^\circ}$$

Prob 16.47 (Cont'd)

The output voltage coefficients:

$$C_0 = 0$$

$$C_{-1} = (10.19/\underline{122.48^\circ})(0.8/\underline{0^\circ}) = 8.15/\underline{122.48^\circ} \text{ V}$$

$$C_1 = 8.15/\underline{-122.48^\circ} \text{ V}$$

$$C_{-2} = (4.30/\underline{-90^\circ})(0.05/\underline{86.19^\circ}) = 0.2287/\underline{-3.81^\circ} \text{ V}$$

$$C_2 = 0.2287/\underline{3.81^\circ} \text{ V}$$

$$C_{-3} = (2.93/\underline{101.98^\circ})(0.03/\underline{87.85^\circ}) = 0.0878/\underline{-170.17^\circ} \text{ V}$$

$$C_3 = 0.0878/\underline{170.17^\circ} \text{ V}$$

$$C_{-4} = (2.15/\underline{-90^\circ})(0.02/\underline{88.47^\circ}) = 0.0458/\underline{-1.53^\circ} \text{ V}$$

$$C_4 = 0.0458/\underline{1.53^\circ} \text{ V}$$

$$\begin{aligned} \text{[d]} \quad V_{\text{rms}} &\cong \sqrt{C_0^2 + 2 \sum_{n=1}^4 |C_n|^2} \cong \sqrt{2 \sum_{n=1}^4 |C_n|^2} \\ &\cong \sqrt{2(8.15^2 + 0.2287^2 + 0.0878^2 + 0.0458^2)} \cong 11.53 \text{ V} \end{aligned}$$

$$P = \frac{(11.53)^2}{250} = 531.95 \text{ mW}$$

Prob 17.1

$$\begin{aligned} \text{P 17.1 [a]} \quad F(\omega) &= \int_{-\tau/2}^{\tau/2} \frac{2A}{\tau} t e^{-j\omega t} dt \\ &= \frac{2A}{\tau} \left[\frac{e^{-j\omega t}}{-\omega^2} (-j\omega t - 1) \right]_{-\tau/2}^{\tau/2} \\ &= \frac{2A}{\omega^2 \tau} \left[e^{-j\omega\tau/2} \left(\frac{j\omega\tau}{2} + 1 \right) - e^{j\omega\tau/2} \left(\frac{-j\omega\tau}{2} + 1 \right) \right] \\ F(\omega) &= \frac{2A}{\omega^2 \tau} \left[e^{-j\omega\tau/2} - e^{j\omega\tau/2} + j \frac{\omega\tau}{2} (e^{-j\omega\tau/2} + e^{j\omega\tau/2}) \right] \\ F(\omega) &= j \frac{2A}{\tau} \left[\frac{\omega\tau \cos(\omega\tau/2) - 2 \sin(\omega\tau/2)}{\omega^2} \right] \end{aligned}$$

[b] Using L'Hopital's rule,

$$\begin{aligned} F(0) &= \lim_{\omega \rightarrow 0} 2A \left[\frac{\omega\tau(\tau/2)[- \sin(\omega\tau/2)] + \tau \cos(\omega\tau/2) - 2(\tau/2) \cos(\omega\tau/2)}{2\omega\tau} \right] \\ &= \lim_{\omega \rightarrow 0} 2A \left[\frac{-\omega\tau(\tau/2) \sin(\omega\tau/2)}{2\omega\tau} \right] \\ &= \lim_{\omega \rightarrow 0} 2A \left[\frac{-\tau \sin(\omega\tau/2)}{4} \right] = 0 \end{aligned}$$

$$\therefore F(0) = 0$$

Prob 17.1 (Cont'd)

[c] When $A = 1$ and $\tau = 1$

$$F(\omega) = j2 \left[\frac{\omega \cos(\omega/2) - 2 \sin(\omega/2)}{\omega^2} \right]$$

$$|F(\omega)| = \left| \frac{2\omega \cos(\omega/2) - 4 \sin(\omega/2)}{\omega^2} \right|$$

$$F(0) = 0$$

$$|F(2)| = \left| \frac{4 \cos 1 - 4 \sin 1}{4} \right| = 0.30$$

$$|F(4)| = \left| \frac{8 \cos 2 - 4 \sin 2}{16} \right| = 0.44$$

$$|F(6)| = \left| \frac{12 \cos 3 - 4 \sin 3}{36} \right| = 0.35$$

$$|F(8)| = \left| \frac{16 \cos 4 - 4 \sin 4}{64} \right| = 0.12$$

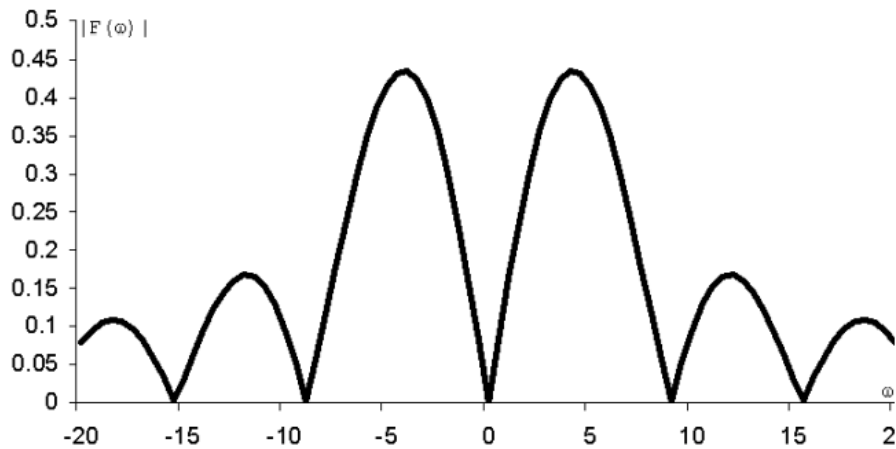
$$|F(9)| = \left| \frac{18 \cos 4.5 - 4 \sin 4.5}{81} \right| \cong 0$$

$$|F(10)| = \left| \frac{20 \cos 5 - 4 \sin 5}{100} \right| = 0.10$$

$$|F(12)| = \left| \frac{24 \cos 6 - 4 \sin 6}{144} \right| = 0.17$$

$$|F(14)| = \left| \frac{28 \cos 7 - 4 \sin 7}{196} \right| = 0.09$$

$$|F(15.5)| = \left| \frac{31 \cos 7.75 - 4 \sin 7.75}{240.25} \right| \cong 0$$



Prob 17.2

$$\text{P 17.2 [a]} \quad F(\omega) = A + \frac{2A}{\omega_o}\omega, \quad -\omega_o/2 \leq \omega \leq 0$$

$$F(\omega) = A - \frac{2A}{\omega_o}\omega, \quad 0 \leq \omega \leq \omega_o/2$$

$$F(\omega) = 0 \quad \text{elsewhere}$$

$$f(t) = \frac{1}{2\pi} \int_{-\omega_o/2}^0 \left(A + \frac{2A}{\omega_o}\omega \right) e^{jt\omega} d\omega$$

$$+ \frac{1}{2\pi} \int_0^{\omega_o/2} \left(A - \frac{2A}{\omega_o}\omega \right) e^{jt\omega} d\omega$$

$$f(t) = \frac{1}{2\pi} \left[\int_{-\omega_o/2}^0 A e^{jt\omega} d\omega + \int_{-\omega_o/2}^0 \frac{2A}{\omega_o} \omega e^{jt\omega} d\omega \right.$$

$$\left. + \int_0^{\omega_o/2} A e^{jt\omega} d\omega - \int_0^{\omega_o/2} \frac{2A}{\omega_o} \omega e^{jt\omega} d\omega \right]$$

$$= \frac{1}{2\pi} [\text{Int1} + \text{Int2} + \text{Int3} - \text{Int4}]$$

$$\text{Int1} = \int_{-\omega_o/2}^0 A e^{jt\omega} d\omega = \frac{A}{jt} (1 - e^{-jt\omega_o/2})$$

$$\text{Int2} = \int_{-\omega_o/2}^0 \frac{2A}{\omega_o} \omega e^{jt\omega} d\omega = \frac{2A}{\omega_o t^2} (1 - j \frac{t\omega_o}{2} e^{-jt\omega_o/2} - e^{-jt\omega_o/2})$$

$$\text{Int3} = \int_0^{\omega_o/2} A e^{jt\omega} d\omega = \frac{A}{jt} (e^{jt\omega_o/2} - 1)$$

$$\text{Int4} = \int_0^{\omega_o/2} \frac{2A}{\omega_o} \omega e^{jt\omega} d\omega = \frac{2A}{\omega_o t^2} (-j \frac{t\omega_o}{2} e^{jt\omega_o/2} + e^{jt\omega_o/2} - 1)$$

$$\text{Int1} + \text{Int3} = \frac{2A}{t} \sin(\omega_o t/2)$$

$$\text{Int2} - \text{Int4} = \frac{4A}{\omega_o t^2} [1 - \cos(\omega_o t/2)] - \frac{2A}{t} \sin(\omega_o t/2)$$

$$\therefore f(t) = \frac{1}{2\pi} \left[\frac{4A}{\omega_o t^2} (1 - \cos(\omega_o t/2)) \right]$$

$$= \frac{2A}{\pi \omega_o t^2} [2 \sin^2(\omega_o t/4)]$$

$$= \frac{4\omega_o A}{\pi \omega_o^2 t^2} \sin^2(\omega_o t/4)$$

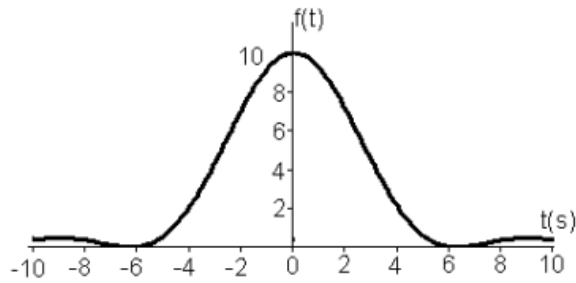
$$= \frac{\omega_o A}{4\pi} \left[\frac{\sin(\omega_o t/4)}{(\omega_o t/4)} \right]^2$$

Prob 17.2 (Cont'd)

$$[b] f(0) = \frac{\omega_o A}{4\pi} (1)^2 = 79.58 \times 10^{-3} \omega_o A$$

$$[c] A = 20\pi; \quad \omega_o = 2 \text{ rad/s}$$

$$f(t) = 10 \left[\frac{\sin(t/2)}{(t/2)} \right]^2$$

**Prob 17.3**

$$P 17.3 \quad [a] F(\omega) = \int_{-2}^2 \left[A \sin\left(\frac{\pi}{2} t\right) \right] e^{-j\omega t} dt = \frac{-j4\pi A}{\pi^2 - 4\omega^2} \sin 2\omega$$

$$[b] F(\omega) = \int_{-\tau/2}^0 \left(\frac{2A}{\tau} t + A \right) e^{-j\omega t} dt + \int_0^{\tau/2} \left(\frac{-2A}{\tau} t + A \right) e^{-j\omega t} dt$$

$$= \frac{4A}{\omega^2 \tau} \left[1 - \cos\left(\frac{\omega\tau}{2}\right) \right]$$

Prob 17.26

P 17.26 [a] $\frac{V_o}{V_g} = H(s) = \frac{4/s}{0.5 + 0.01s + 4/s}$

$$H(s) = \frac{400}{s^2 + 50s + 400} = \frac{400}{(s + 10)(s + 40)}$$

$$H(j\omega) = \frac{400}{(j\omega + 10)(j\omega + 40)}$$

$$V_g(\omega) = \frac{6}{j\omega}$$

$$V_o(\omega) = V_g(\omega)H(j\omega) = \frac{2400}{j\omega(j\omega + 10)(j\omega + 40)}$$

$$V_o(\omega) = \frac{K_1}{j\omega} + \frac{K_2}{j\omega + 10} + \frac{K_3}{j\omega + 40}$$

$$K_1 = \frac{2400}{400} = 6; \quad K_2 = \frac{2400}{(-10)(30)} = -8$$

$$K_3 = \frac{2400}{(-40)(-30)} = 2$$

$$V_o(\omega) = \frac{6}{j\omega} - \frac{8}{j\omega + 10} + \frac{2}{j\omega + 40}$$

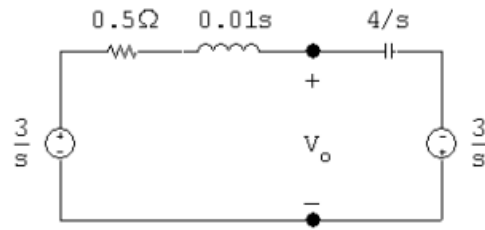
$$v_o(t) = 3\text{sgn}(t) - 8e^{-10t}u(t) + 2e^{-40t}u(t) \text{ V}$$

[b] $v_o(0^-) = -3 \text{ V}$

[c] $v_o(0^+) = 3 - 8 + 2 = -3 \text{ V}$

Prob 17.26 (Cont'd)

[d] For $t \geq 0^+$:



$$\frac{V_o - 3/s}{0.5 + 0.01s} + \frac{(V_o + 3/s)s}{4} = 0$$

$$V_o \left[\frac{100}{s + 50} + \frac{s}{4} \right] = \frac{300}{s(s + 50)} - 0.75$$

$$V_o = \frac{1200 - 3s^2 - 150s}{s(s + 10)(s + 40)} = \frac{K_1}{s} + \frac{K_2}{s + 10} + \frac{K_3}{s + 40}$$

$$K_1 = \frac{1200}{400} = 3; \quad K_2 = \frac{1200 - 300 + 1500}{(-10)(30)} = -8$$

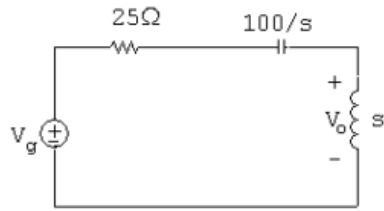
$$K_3 = \frac{1200 - 4800 + 6000}{(-40)(-30)} = 2$$

$$v_o(t) = (3 - 8e^{-10t} + 2e^{-40t})u(t) \text{ V}$$

[e] Yes.

Prob 17.35

P 17.35 [a]



$$V_o = \frac{V_g s}{25 + (100/s) + s} = \frac{V_g s^2}{s^2 + 25s + 100}$$

$$H(s) = \frac{V_o}{V_g} = \frac{s^2}{(s+5)(s+20)}; \quad H(j\omega) = \frac{(j\omega)^2}{(j\omega+5)(j\omega+20)}$$

$$v_g = 25i_g = 450e^{10t}u(-t) - 450e^{-10t}u(t) \text{ V}$$

$$V_g = \frac{450}{-j\omega+10} - \frac{450}{j\omega+10}$$

$$V_o(\omega) = H(j\omega)V_g = \frac{450(j\omega)^2}{(-j\omega+10)(j\omega+5)(j\omega+20)}$$

$$+ \frac{-450(j\omega)^2}{(j\omega+10)(j\omega+5)(j\omega+20)}$$

$$= \frac{K_1}{-j\omega+10} + \frac{K_2}{j\omega+5} + \frac{K_3}{j\omega+20} + \frac{K_4}{j\omega+5} + \frac{K_5}{j\omega+10} + \frac{K_6}{j\omega+20}$$

$$K_1 = \frac{450(100)}{(15)(30)} = 100 \quad K_4 = \frac{-450(25)}{(5)(15)} = -150$$

$$K_2 = \frac{450(25)}{(15)(15)} = 50 \quad K_5 = \frac{-450(100)}{(-5)(10)} = 900$$

$$K_3 = \frac{450(400)}{(30)(-15)} = -400 \quad K_6 = \frac{-450(400)}{(-15)(-10)} = -1200$$

$$V_o(\omega) = \frac{100}{-j\omega+10} + \frac{-100}{j\omega+5} + \frac{-1600}{j\omega+20} + \frac{900}{j\omega+10}$$

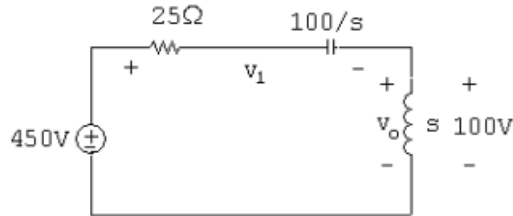
$$v_o = 100e^{10t}u(-t) + [900e^{-10t} - 100e^{-5t} - 1600e^{-20t}]u(t) \text{ V}$$

Prob 17.35 (Cont'd)

[b] $v_o(0^-) = 100 \text{ V}$

[c] $v_o(0^+) = 900 - 100 - 1600 = -800 \text{ V}$

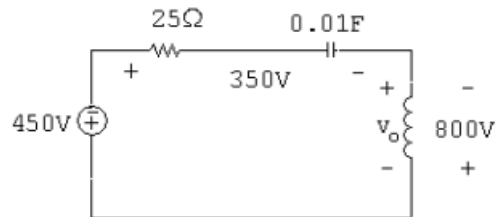
[d] At $t = 0^-$ the circuit is



Therefore, the solution predicts $v_1(0^-)$ will be 350 V.

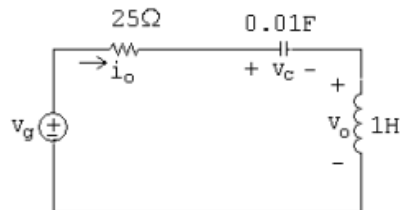
Now $v_1(0^+) = v_1(0^-)$ because the inductor will not let the current in the 25Ω resistor change instantaneously, and the capacitor will not let the voltage across the 0.01 F capacitor change instantaneously.

At $t = 0^+$ the circuit is



From the circuit at $t = 0^+$ we see that v_o must be -800 V , which is consistent with the solution for v_o obtained in part (a).

It is informative to solve for either the current in the circuit or the voltage across the capacitor and note the solutions for i_o and v_C are consistent with the solution for v_o



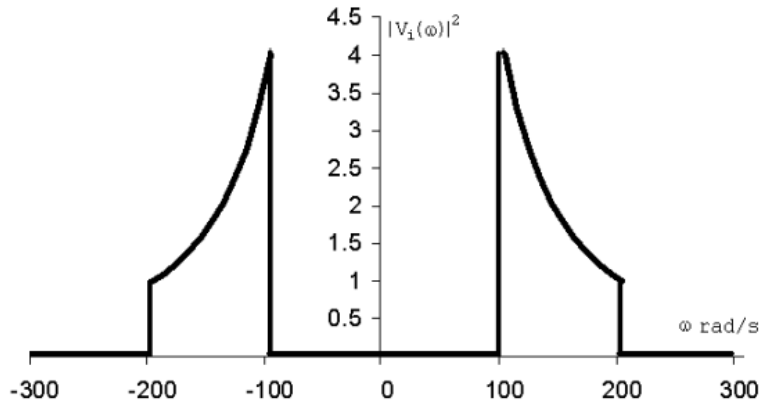
The solutions are

$$i_o = 10e^{10t}u(-t) + [20e^{-5t} + 80e^{-20t} - 90e^{-10t}]u(t) \text{ A}$$

$$v_C = 100e^{10t}u(-t) + [900e^{-10t} - 400e^{-5t} - 400e^{-20t}]u(t) \text{ V}$$

Prob 17.41

P 17.41 [a] $|V_i(\omega)|^2 = \frac{4 \times 10^4}{\omega^2}$; $|V_i(100)|^2 = \frac{4 \times 10^4}{100^2} = 4$; $|V_i(200)|^2 = \frac{4 \times 10^4}{200^2} = 1$



[b] $V_o = \frac{V_i R}{R + (1/sC)} = \frac{sRCV_i}{RCs + 1}$

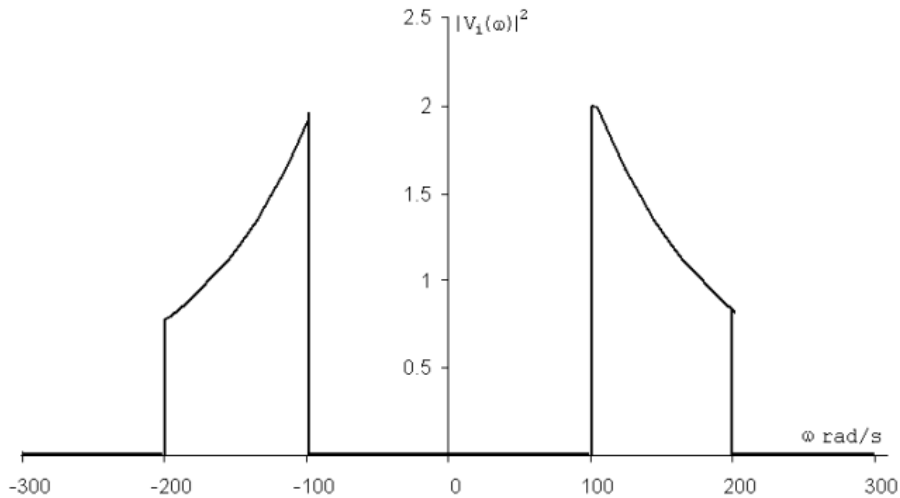
$H(s) = \frac{V_o}{V_i} = \frac{s}{s + (1/RC)}$; $\frac{1}{RC} = \frac{10^6 10^{-3}}{(0.5)(20)} = \frac{1000}{10} = 100$

$H(j\omega) = \frac{j\omega}{j\omega + 100}$

$|V_o(\omega)| = \frac{200}{|\omega|} \cdot \frac{|\omega|}{\sqrt{\omega^2 + 10^4}} = \frac{200}{\sqrt{\omega^2 + 10^4}}$

$|V_o(\omega)|^2 = \frac{4 \times 10^4}{\omega^2 + 10^4}$, $100 \leq \omega \leq 200$ rad/s; $|V_o(\omega)|^2 = 0$, elsewhere

$|V_o(100)|^2 = \frac{4 \times 10^4}{10^4 + 10^4} = 2$; $|V_o(200)|^2 = \frac{4 \times 10^4}{5 \times 10^4} = 0.8$



Prob 17.41 (Cont'd)

$$\begin{aligned} \text{[c]} \quad W_{1\Omega} &= \frac{1}{\pi} \int_{100}^{200} \frac{4 \times 10^4}{\omega^2} d\omega = \frac{4 \times 10^4}{\pi} \left[-\frac{1}{\omega} \right]_{100}^{200} \\ &= \frac{4 \times 10^4}{\pi} \left[\frac{1}{100} - \frac{1}{200} \right] = \frac{200}{\pi} \cong 63.66 \text{ J} \end{aligned}$$

$$\begin{aligned} \text{[d]} \quad W_{1\Omega} &= \frac{1}{\pi} \int_{100}^{200} \frac{4 \times 10^4}{\omega^2 + 10^4} d\omega = \frac{4 \times 10^4}{\pi} \cdot \tan^{-1} \frac{\omega}{100} \Big|_{100}^{200} \\ &= \frac{400}{\pi} [\tan^{-1} 2 - \tan^{-1} 1] \cong 40.97 \text{ J} \end{aligned}$$