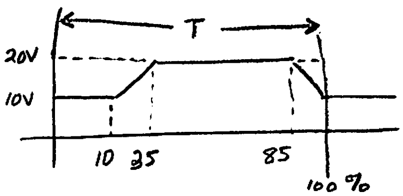
EENG382 QUIZ #1 (Rev 01)

Problem #1

A periodic voltage waveform can be described as follows: The voltage transistions between a "low" voltage, V_L, and a "high" voltage, V_H, via a linear ramp. The voltage is low 20% of the time and high 50% of the time with the remaining 30% split evenly between the two transitional ramps. If V_L=10V and V_H=20V, what are the average and rms voltages of this waveform?



$$v(t) = \begin{cases} v(t+T) & t \le 0 \\ 10V & 0 \le t \le 20\%T \\ 10V + \frac{10V}{15\%T}(t - 10\%T) & 20\%T \le t \le 35\%T \\ 20V & 35\%T \le t \le 85\%T \\ 20V - \frac{10V}{15\%T}(t - 85\%T) & 85\%T \le t \le 100\%T \\ v(t-T) & 100\%T \le t \end{cases}$$

$$V_{AVG} = \frac{1}{T}\int_0^T v(t)\,dt \Rightarrow \frac{AREA}{T} = \frac{10V \cdot 20\%T + 15V \cdot 15\%T + 20V \cdot 50\%T + 15V \cdot 15\%T}{T}$$

$$= 2V + 2.25V + 10V + 2.25V$$

$$\underline{\underline{V_{AVG} = 16.5V}} \leftarrow \text{(1a)}$$

$$V_{RMS} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt} \Rightarrow \sqrt{\frac{\text{AREA UNDER } v^2(t)}{T}}$$

$$\text{AREA UNDER } v^2(t) = (10V)^2 \cdot 20\%T + (20V)^2 \cdot 50\%T + 2\left[\int_0^{15\%T}\left(10V + \frac{10V}{15\%T}t\right)^2 dt\right]$$

$$\int_0^{15\%T}\left(10V + \frac{10V}{15\%T}t\right)^2 dt = \int_0^{15\%T}\left(100V^2 + \frac{200V^2}{15\%T}t + \frac{100V^2}{(15\%T)^2}t^2\right) dt$$

$$= 100V^2(15\%T) + \frac{200V^2}{15\%T}\cdot\frac{(15\%T)^2}{2} + \frac{100V^2}{(15\%T)^2}\cdot\frac{(15\%T)^3}{3}$$

$$= 100V^2(15\%T)\left[1 + 1 + \frac{1}{3}\right]$$

$$\text{AREA UNDER } v^2(t) = 100V^2\left[20\% + 4 \cdot 50\% + 2[2.333]15\%\right]T$$

$$= 100V^2(3.100)T$$

$$V_{RMS} = \sqrt{\frac{100V^2(3.100)T}{T}}$$

$$= 10V\sqrt{3.100}$$

$$\underline{\underline{V_{RMS} = 17.61V}} \leftarrow \text{(1b)}$$

Problem #2

10.18 a) Find the average power, the reactive power, and the apparent power supplied by the voltage source in the circuit in Fig. P10.18 if $v_g = 40 \cos 10^6 t$ V.

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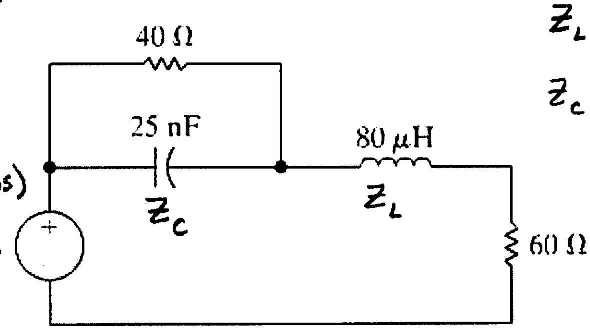
b) Check your answer in (a) by showing $P_{dev} = \sum P_{abs}$.

c) Check your answer in (a) by showing $Q_{dev} = \sum Q_{abs}$.

Figure P10.18



$$Z_L = j\omega L = j \cdot 10^6 \tfrac{r}{s} \cdot 80\mu H = j80\Omega$$

$$Z_C = \frac{1}{j\omega C} = \frac{1}{j \cdot 10^6 \frac{r}{s} \cdot 25nF} = -j40\Omega$$

$\vec{V}_g = \frac{40V}{\sqrt{2}}$ (RMS)

$\omega = 10^6 \frac{r}{s}$

$$Z_T = (40\Omega \,||\, -j40\Omega) + (60\Omega + j80\Omega)$$

$$= \frac{(40\Omega)(-j40\Omega)(1+j)}{40\Omega(1-j)(1+j)} = \frac{40\Omega}{2}(1-j) + 60\Omega + j80\Omega = 80\Omega + j60\Omega$$

$$\vec{S} = \frac{V^2}{Z_T^*} = \frac{\left(\frac{40V}{\sqrt{2}}\right)^2}{(80\Omega - j60\Omega)} = \frac{1600V^2}{2 \cdot 20\Omega(4-j3)} = \frac{40VA \cdot (4+j3)}{(4-j3)(4+j3)} = \frac{40VA}{25}(4+j3)$$

$\vec{S} = 6.4W + j4.8\,VAR$

$P_{AVG} = 6.4W$
$Q = 4.8VAR$
$S = 8.0\,VA$ (2a)

$$I_{TOT} = \frac{\frac{40V}{\sqrt{2}}}{20\Omega(4+3j)} = \frac{2}{\sqrt{2}}A \frac{1\,(4-3j)}{(4+3j)(4-3j)} = \frac{2(4-j3)A}{25\sqrt{2}}; \; |I_{TOT}| = \frac{0.4}{\sqrt{2}}A$$

$$I_{40\Omega} = I_{TO}\frac{Z_C}{40\Omega + Z_C} = \frac{2(4-j3)A}{25\sqrt{2}}\left(\frac{-j40\Omega}{40\Omega - j40\Omega}\right) = \frac{2(4-j3)(-j)(1+j)}{25\cdot\sqrt{2}(1-j)(1+j)}A = \frac{1-j7}{25\sqrt{2}}A; \; |I_{40\Omega}| = \frac{\sqrt{50}}{25\cdot\sqrt{2}}A = \frac{1}{5}A$$

$jQ_{Z_L} = I^2 Z_L = \left(\frac{0.4A}{\sqrt{2}}\right)^2(j80\Omega) = j6.4\,VAR$

$P_{60\Omega} = I^2 R = \left(\frac{0.4A}{\sqrt{2}}\right)^2 \cdot 60\Omega = 4.8W$

$V_{Z_C} = I_{40\Omega} \cdot 40\Omega = (0.2A)(40\Omega) = 8V$

$P_{40\Omega} = \left(\frac{1A}{5}\right)^2 \cdot 40\Omega = 1.6W$

$jQ_{Z_C} = \frac{V_{Z_C}^2}{Z_{Z_C}^*} = \frac{(8V)^2}{j40\Omega} = -j1.6\,VAR$

$P_{TOT} = P_{60\Omega} + P_{40\Omega} = 4.8W + 1.6W = 6.4W$ (2b)

$Q_{TOT} = Q_{Z_L} + Q_{Z_C} = 6.4VAR - 1.6VAR = 4.8VAR$ (2c)

Problem #3

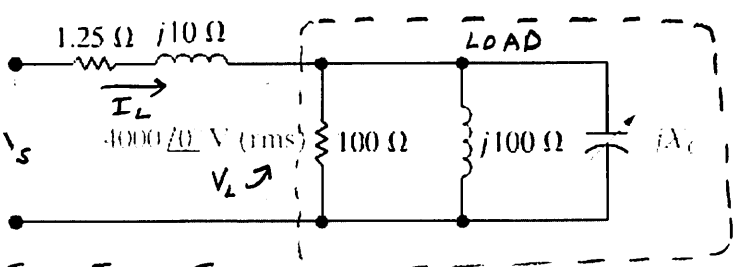
10.54 The sending-end voltage in the circuit seen in Fig. P10.54 is adjusted so that the rms value of the load voltage is always 4000 V. The variable capacitor is adjusted until the average power dissipated in the line resistance is minimum.

a) If the frequency of the sinusoidal source is 60 Hz, what is the value of the capacitance in microfarads?

b) If the capacitor is removed from the circuit, what percentage increase in the magnitude of $\mathbf{V}_s$ is necessary to maintain 4000 V at the load?

c) If the capacitor is removed from the circuit, what is the percentage increase in line loss?

Figure P10.54



$I_{LINE} = I_{LOAD} = I_L$

- TO MINIMIZE POWER DISSIPATION IN LINE, WE MUST MINIMIZE I_L
- SINCE V_L IS CONSTANT, $\vec{I}_L = \dfrac{4000V\angle 0^\circ}{100\Omega} + \dfrac{4000V\angle 0^\circ}{X_L} = \underset{\text{FIXED}}{\underset{\uparrow}{40A\angle 0^\circ}} + \underset{\text{VARIABLE.}}{\underset{\uparrow}{\dfrac{4000V\angle 0^\circ}{X_L}}}$
- WE NEED TO MAKE THE REACTIVE PART OF THE LOAD VANISH SO THAT REACTIVE POWER IN THE LOAD REMAINS LOCAL.

$$\therefore -j\frac{1}{\omega C} = -j100\Omega \Rightarrow C = \frac{1}{\omega\, 100\Omega} = \frac{F}{2\pi f\cdot 100\Omega} = \frac{\text{SEC}}{2\pi\cdot 60\ \ 100\Omega} = \underline{\underline{26.5\mu F}} \leftarrow (3a)$$

w/CAP: $P_{LINE_0} = (40A)^2 R$; $\vec{V}_s = 4000V\angle 0^\circ + (40A\angle 0^\circ)(1.25\Omega + j10\Omega)$

$= (4050 + j400)V$; $|\vec{V}_s| = 4070V$

w/o CAP: $Z_L = \dfrac{(100\Omega)(j100\Omega)}{100\Omega(1+j)} = \dfrac{100\Omega}{\sqrt{2}}\angle 45^\circ$

$I_L = \dfrac{4000V\angle 0^\circ}{\frac{100\Omega}{\sqrt{2}}\angle 45^\circ} = \sqrt{2}\cdot 40A\angle -45^\circ$

$P_{LINE} = (\sqrt{2}\cdot 40A)^2 R = 2\cdot(40A)^2 R = 2P_{LINE_0}$; $\vec{V}_s = 4000V\angle 0^\circ + \sqrt{2}\,40A\left(\frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2}\right)(1.25\Omega + j10\Omega)$

$= 4000V + 50V + 400V + j(400V - 50V)$

$= (4450 + j350)V$

$|\vec{V}_s| = 4464V$

$\underline{\underline{\% \text{ INCREASE IN } V_s = \dfrac{4464V - 4070V}{4070V} = 9.68\%}} \leftarrow (3b)$

$\underline{\underline{\% \text{ INCREASE IN } P_{LINE} = \dfrac{2P_{LINE_0} - P_{LINE_0}}{P_{LINE_0}} = 100\%}} \leftarrow (3c)$

Problem #4

Find the one-sided Laplace transform of the following function beginning with the definition of the one-sided Laplace transform.

$e^{-at}\cos\omega t$ (damped cosine)

$$\mathcal{L}\{e^{-at}\cos(\omega t)\} = \int_{0^-}^{\infty} e^{-at}\cos(\omega t)\, e^{-st}\, dt$$

$$= \int_{0^-}^{\infty} \frac{1}{2}\left(e^{j\omega t} + e^{-j\omega t}\right) e^{-(s+a)t}\, dt$$

$$= \frac{1}{2}\left[\int_{0^-}^{\infty} e^{-(s+a-j\omega)t}\, dt + \int_{0^-}^{\infty} e^{-(s+a+j\omega)t}\, dt\right]$$

$$= \frac{1}{2}\left[\frac{-1}{s+a-j\omega}\underbrace{e^{-(s+a-j\omega)t}\Big|_{0^-}^{\infty}}_{(0-1)} + \frac{-1}{s+a+j\omega}\underbrace{e^{-(s+a-j\omega)t}\Big|_{0^-}^{\infty}}_{(0-1)}\right]$$

$$= \frac{1}{2}\left[\frac{1}{s+a-j\omega} + \frac{1}{s+a+j\omega}\right]$$

$$= \frac{1}{2}\,\frac{(s+a+j\omega)+(s+a-j\omega)}{(s+a)^2+\omega^2}$$

$$= \frac{1}{2}\,\frac{2(s+a)}{(s+a)^2+\omega^2}$$

$$\therefore \mathcal{L}\{e^{-at}\cos(\omega t)\} = \frac{(s+a)}{(s+a)^2+\omega^2} \qquad (4)$$

Problem #5

Prove/derive the following operation Laplace transform

$f(at), a > 0$ $\qquad\qquad$ $\frac{1}{a}F\left(\frac{s}{a}\right)$

$$\mathcal{L}\{f(at)\} = \int_{0^-}^{\infty} f(at)\, e^{-st}\, dt$$

$\tau = at \Rightarrow t = \frac{\tau}{a}$

$d\tau = a\,dt \quad dt = \frac{d\tau}{a}$

(LIMITS ARE UNCHANGED)

$$= \int_{0^-}^{\infty} \frac{1}{a} f(\tau)\, e^{-\left(\frac{s}{a}\right)\tau}\, d\tau$$

$$\mathcal{L}\{f(at)\} = \frac{1}{a}F\left(\frac{s}{a}\right) \longleftarrow$$

(5)