PROBLEM #12.54

12.54 a) Use phasor circuit analysis techniques from Chapter 9 to determine the steady-state expression for the inductor current in Fig. 12.18.

b) How does your result in part (a) compare to the complete response as given in Eq. 12.109?

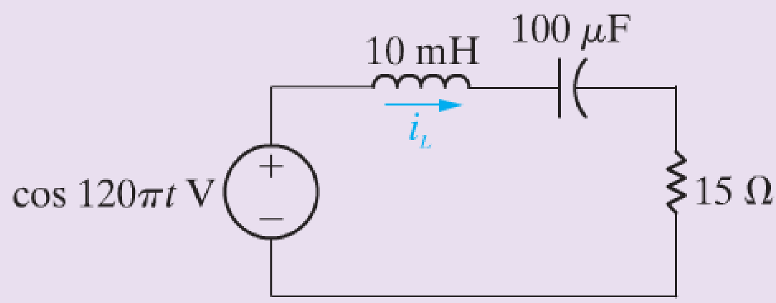

Figure 12.18 ▲ A series *RLC* circuit with a 60 Hz sinusoidal source.

PART (a)

$$Z_L = j\omega L = j120\pi\ ^r/_s \cdot 10\,mH = 3.770\,\Omega$$
$$Z_C = \frac{1}{j\omega C} = -j\frac{1}{120\pi}\cdot\frac{1}{100\mu F} = -j\,26.53\,\Omega$$
$$V_S = 1V\angle 0^\circ$$

$$I_L = \frac{V_S}{R+Z_L+Z_C} = \frac{1V\angle 0^\circ}{15\Omega + j(3.770\Omega - 26.53\Omega)} = \frac{1V\angle 0^\circ}{27.25\Omega\angle -56.61^\circ}$$
$$= \frac{1V\angle 0^\circ}{27.25\Omega\angle -56.61^\circ} = 36.70\,mA\angle 56.61^\circ$$

$$\underline{\underline{i_L(t) = 36.7\,mA\cos\left((120\pi\ ^r/_s)t + 56.6^\circ\right)}} \leftarrow \text{(\#1254a)}$$

PART (b)

$$\text{EQ 12.109: } i_L(t) = 147.14\,mA\,e^{-(750\,/s)t}\cos\left((661.44\,/s)t - 97.89^\circ\right)$$
$$+ 36.69\,mA\cos\left((120\pi\ ^r/_s)t + 56.61^\circ\right)$$

AS $t\to\infty$, THE EXPONENTIAL IN THE FIRST TERM DRIVES IT TO ZERO, LEAVING ONLY THE SECOND TERM WHICH MATCHES THE RESULTS FROM PART (a).

$$\underline{\underline{\therefore \lim_{t\to\infty}(\text{EQ 12.109}) = (\#12.54a)}} \leftarrow \text{(\#12.54b)}$$

PROBLEM #12.55

12.55 Find the maximum magnitude of the sinusoidal source in Fig. 12.18 such that the complete response of the inductor current does not exceed the 40 mA current rating at $t = 1$ ms.

ACCORDING TO THE AUTHOR'S ANALYS ON PP 458-9, THE MAXIMUM INSTANTANEOUS CURRENT IS 42.6mA. SINCE THIS IS A LINEAR CIRCUIT, WE KNOW THAT THE RESPONSE IS DIRECTLY PROPORTIONAL TO THE INPUT. HENCE, TO KEEP THE RESPONSE BELOW 40mA, WE SIMPLY NEED TO SCALE THE 1V SIGNAL AMPLITUDE DOWN PROPORTIONATELY.

$$\underline{\underline{V_{IN} = 1V \cdot \frac{40mA}{42.6mA} = 0.9390V}} \leftarrow \text{(\#12.55)}$$

HOWEVER, IF WE EVALUATE EQUATION 12.109 AT $t = 1ms$, WE GET $i_L(t = 1ms) = 42.26mA$. USING THIS VALUE, WE GET

$$\underline{\underline{V_{IN} = 1V \cdot \frac{40mA}{42.26mA} = 0.947V}} \leftarrow$$

PROBLEM #12.56

12.56 Suppose the input to the circuit in Fig 12.18 is a damped ramp of the form Kte^{-100t} V. Find the largest value of K such that the inductor current does not exceed the 40 mA current rating.

$$v_{in}(t) = Kt\,e^{-\frac{t}{\tau_o}}\,u(t) \Rightarrow V_{IN}(s) = K\frac{1}{(s+1/\tau_o)^2} \quad (\tau_o = 10\,ms)$$

$$v_{in}(t) = v_r(t) + v_c(t) + v_L(t) = R\cdot i(t) + V_c(t{=}0) + \frac{1}{C}\int_0^t i(t)\,dt + L\frac{di(t)}{dt}$$

THE PROBLEM IMPLIES THAT THERE IS NO INITIAL VOLTAGE ON THE CAPACITOR AND REQUIRES THERE BE NO INITIAL CURRENT IN THE INDUCTOR. THEREFORE

$$v_{IN}(t) = R\cdot i(t) + \frac{1}{C}\int_0^t i(t)\,dt + L\frac{di(t)}{dt}$$

$$V_{IN}(s) = R\,I(s) + \frac{1}{Cs}I(s) + Ls\,I(s) - \cancelto{0}{L\,i(t{=}0)}$$

$$\frac{K}{(s+1/\tau_o)^2} = I(s)\cdot\left(\frac{L}{s}\right)\left(s^2 + \frac{s}{(L/R)} + \frac{1}{LC}\right)$$

$$I(s) = \left(\frac{K}{L}\right)\frac{s}{(s+1/\tau_o)^2\left(s^2+\frac{s}{(L/R)}+\frac{1}{LC}\right)}$$

ROOTS OF QUADRATIC: $-\frac{1}{2}\left[\left(\frac{R}{L}\right) \pm \sqrt{\left(\frac{R}{L}\right)^2 - \left(\frac{4}{LC}\right)}\right]$

COMPLEX ROOTS IF $\frac{4}{LC} > \left(\frac{R}{L}\right)^2 \Rightarrow 4\frac{L}{R} > RC$

$L/R = \frac{10\,mH}{15\,\Omega} = 0.667\,ms$; $RC = 15\,\Omega\cdot 100\,\mu S = 1.5\,ms$

$\therefore$ COMPLEX ROOTS OF $-\frac{1}{2}\frac{15\,\Omega}{10\,mH} \pm \frac{1}{2}\sqrt{\left(\frac{15\,\Omega}{10\,mH}\right)^2 - \frac{4}{(10\,mH\cdot 100\,\mu F)}}$

$$\text{roots} = -750\,r/s \pm j661.44\,r/s = -(\alpha \mp \beta)$$

$$I(s) = \left(\frac{K}{L}\right)\left[\frac{A}{(s+1/\tau_o)} + \frac{B}{(s+1/\tau_o)^2} + \frac{C}{(s+\alpha-j\beta)} + \frac{C^*}{(s+\alpha+j\beta)}\right]$$

$$B = \left.\frac{s}{s^2+\frac{s}{(L/R)}+\frac{1}{LC}}\right|_{s=-\frac{1}{\tau_o}} = \frac{-1/\tau_o}{(1/\tau_o)^2 - \left(\frac{1}{\tau_o(L/R)}\right) + \frac{1}{LC}}$$

$$B = -\tau_o\left(\frac{1}{1-\frac{\tau_o}{(L/R)}+\frac{\tau_o^2}{LC}}\right) = -10\,ms\left(\frac{1}{1-\frac{10\,ms}{0.667\,ms}+\frac{100\,(ms)^2}{1\,(ms)^2}}\right)$$

$$\underline{B = -0.11628\,ms}$$

TOPS 35500

PROBLEM #12.56 (CONT'D)

$$A = \frac{d}{ds}\left(\frac{s}{s^2+\frac{s}{(L/R)}+\frac{1}{LC}}\right)\Bigg|_{s=-\frac{1}{\tau_o}}$$

$$= \left[\frac{1}{s^2+\frac{s}{(L/R)}+\frac{1}{LC}} - \frac{s\left(2s+\frac{1}{(L/R)}\right)}{\left(s^2+\frac{s}{(L/R)}+\frac{1}{LC}\right)^2}\right]_{s=-\frac{1}{\tau_o}}$$

$$= \frac{s^2+\frac{s}{(L/R)}+\frac{1}{LC} - 2s^2 - \frac{s}{(L/R)}}{\left(s^2+\frac{s}{(L/R)}+\frac{1}{LC}\right)^2}\Bigg|_{s=-\frac{1}{\tau_o}}$$

$$= \frac{-s^2+\frac{1}{LC}}{\left(s^2+\frac{s}{(L/R)}+\frac{1}{LC}\right)^2}\Bigg|_{s=-\frac{1}{\tau_o}}$$

$$= \frac{-\left(\frac{1}{\tau_o}\right)^2+\frac{1}{LC}}{\left[\left(\frac{1}{\tau_o}\right)^2 - \frac{1}{\tau_o(L/R)}+\frac{1}{LC}\right]^2}$$

$$= \frac{\left(\frac{1}{\tau_o}\right)^2\left(\frac{\tau_o^2}{LC}-1\right)}{\left(\frac{1}{\tau_o}\right)^4\left[1-\frac{\tau_o}{(L/R)}+\frac{\tau_o^2}{LC}\right]^2}$$

$$= \tau_o^2\,\frac{\left(\frac{\tau_o^2}{LC}-1\right)}{\left[1-\frac{\tau_o}{(L/R)}+\frac{\tau_o^2}{LC}\right]^2}$$

$$= (10ms)^2\,\frac{\left(\frac{(10ms)^2}{(1ms)^2}-1\right)}{\left[1-\frac{10ms}{0.667ms}+\frac{(10ms)^2}{(1ms)}\right]^2}$$

$$= (10ms)^2\,\frac{99}{(1-15+100)^2} = (10ms)^2\,\frac{99}{86^2}$$

$$\underline{A = (1.1570ms)^2}$$

$$C = \frac{s}{(s+1/\tau_o)^2(s+\alpha+j\beta)}\Bigg|_{s=-\alpha+j\beta}$$

$$= \frac{(-\alpha+j\beta)}{(1/\tau_o-\alpha+j\beta)^2(j2\beta)}$$

$$= \frac{(-750-j661.44)\,r/s}{(100\,r/s-750\,r/s-j661.44\,r/s)^2(j2(-661.44\,r/s))}$$

$$\underline{C = (0.9375ms \angle -69.79^\circ)^2}$$

PROBLEM #12.56 (CONT'D)

$$i(t) = \frac{K}{L}\left[Ae^{-\frac{t}{\tau_0}} + Bte^{-\frac{t}{\tau_0}} + 2|C|e^{-\alpha t}\cos(\beta t - \measuredangle C)\right]$$

TO FIND $i(t)_{max}$, WE COULD TAKE THE DERIVATIVE OF $i(t)$ AND SET IT TO ZERO. HOWEVER, THIS IS CLEARLY AN EQUATION THAT WILL NOT YIELD AN ANALYTIC ANSWER AND WILL THEREFORE REQUIRE AN ITERATIVE APPROACH. THAT BEING THE CASE, IT IS JUST AS REASONABLE TO USE AN ITERATIVE APPROACH TO FIND max $i(t)$ DIRECTLY).

PLOTTING $i(t)$ USING EXCEL YIELDS SHOWS THAT THE PEAK CURRENT FOR $K = 1\,V/s$ IS $68.0\,\mu A$ AND OCCURS JUST PRIOR TO $t = 3ms$.

SCALING THIS TO A PEAK CURRENT OF 40mA YIELDS

$$K = 1\,V/s \cdot \frac{40\,mA}{68.0\,\mu A} = 588\,V/s$$

$$\underline{\underline{K = 0.588\ V/ms}}$$ (#12.56)