EENG382 HW07 – AUTHOR'S SOLUTIONS

NOTE: I have not yet verified that the author's solutions are, in fact, correct.

<u>Prob 14.46</u>

P 14.46 [**a**] Use the cutoff frequencies to calculate the bandwidth:

$$\omega_{c1} = 2\pi(697) = 4379.38 \text{ rad/s} \qquad \omega_{c2} = 2\pi(941) = 5912.48 \text{ rad/s}$$

Thus $\quad \beta = \omega_{c2} - \omega_{c1} = 1533.10$ rad/s

Calculate inductance using Eq. (14.32) and capacitance using Eq. (14.31):

$$L = \frac{R}{\beta} = \frac{600}{1533.10} = 0.39\,\text{H}$$

$$C = \frac{1}{L\omega_{c1}\omega_{c2}} = \frac{1}{(0.39)(4379.38)(5912.48)} = 0.10\,\mu\text{F}$$

[**b**] At the outermost two frequencies in the low-frequency group (687 Hz and 941 Hz) the amplitudes are

$$|\text{V}_{697\text{Hz}}| = |V_{941\text{Hz}}| = \frac{|V_{\text{peak}}|}{\sqrt{2}} = 0.707|V_{\text{peak}}|$$

because these are cutoff frequencies. We calculate the amplitudes at the other two low frequencies using Eq. (14.32):

$$|V| = (|V_{\text{peak}}|)(|H(j\omega)|) = |V_{\text{peak}}|\frac{\omega\beta}{\sqrt{(\omega_o^2 - \omega^2)^2 + (\omega\beta)^2}}$$

Therefore

$$|V_{770\text{Hz}}| = |V_{\text{peak}}| = \frac{(4838.05)(1533.10)}{\sqrt{(5088.52^2 - 4838.05^2)^2 + [(4838.05)(1533.10)]^2}}$$

$$= 0.948|V_{\text{peak}}|$$

and

$$|V_{852\text{Hz}}| = |V_{\text{peak}}| = \frac{(5353.27)(1533.10)}{\sqrt{(5088.52^2 - 5353.27^2)^2 + [(5353.27)(1533.10)]^2}}$$

$$= 0.948|V_{\text{peak}}|$$

Prob 14.46 (Cont'd)

It is not a coincidence that these two magnitudes are the same. The frequencies in both bands of the DTMF system were carefully chosen to produce this type of predictable behavior with linear filters. In other words, the frequencies were chosen to be equally far apart with respect to the response produced by a linear filter. Most musical scales consist of tones designed with this dame property – note intervals are selected to place the notes equally far apart. That is why the DTMF tones remind us of musical notes! Unlike musical scales, DTMF frequencies were selected to be harmonically unrelated, to lower the risk of misidentifying a tone's frequency if the circuit elements are not perfectly linear.

[c] The high-band frequency closest to the low-frequency band is 1209 Hz. The amplitude of a tone with this frequency is

$$|V_{1209\mathrm{Hz}}| = |V_{\mathrm{peak}}| = \frac{(7596.37)(1533.10)}{\sqrt{(5088.52^2 - 7596.37^2)^2 + [(7596.37)(1533.10)]^2}}$$

$$= 0.344|V_{\mathrm{peak}}|$$

This is less than one half the amplitude of the signals with the low-band cutoff frequencies, ensuring adequate separation of the bands.

Prob 14.47

P 14.47 The cutoff frequencies and bandwidth are

$$\omega_{c_1} = 2\pi(1209) = 7596\,\mathrm{rad/s}$$

$$\omega_{c_2} = 2\pi(1633) = 10.26\,\mathrm{krad/s}$$

$$\beta = \omega_{c_2} - \omega_{c_1} = 2664\,\mathrm{rad/s}$$

Telephone circuits always have $R = 600\,\Omega$. Therefore, the filters inductance and capacitance values are

$$L = \frac{R}{\beta} = \frac{600}{2664} = 0.225\,\mathrm{H}$$

$$C = \frac{1}{\omega_{c_1}\omega_{c_2}L} = 0.057\,\mu\mathrm{F}$$

At the highest of the low-band frequencies, 941 Hz, the amplitude is

$$|V_\omega| = |V_{\mathrm{peak}}|\frac{\omega\beta}{\sqrt{(\omega_o^2 - \omega^2)^2 + \omega^2\beta^2}}$$

where $\qquad \omega_o = \sqrt{\omega_{c_1}\omega_{c_2}}$. Thus,

$$|V_\omega| = \frac{|V_{\mathrm{peak}}|(5912)(2664)}{\sqrt{[(8828)^2 - (5912)^2]^2 + [(5912)(2664)]^2}}$$

$$= 0.344\ |V_{\mathrm{peak}}|$$

Again it is not coincidental that this result is the same as the response of tl low-band filter to the lowest of the high-band frequencies.

Prob 14.48

P 14.48 From Problem 14.46 the response to the largest of the DTMF low-band tones is $0.948|V_{\rm peak}|$. The response to the 20 Hz tone is

$$|V_{\rm 20Hz}| = \frac{|V_{\rm peak}|(125.6)(1533)}{[(5089^2 - 125.6^2)^2 + [(125.6)(1533)]^2]^{1/2}}$$

$$= 0.00744|V_{\rm peak}|$$

$$\therefore \quad \frac{|V_{\rm 20Hz}|}{|V_{\rm 770Hz}|} = \frac{|V_{\rm 20Hz}|}{|V_{\rm 852Hz}|} = \frac{0.00744|V_{\rm peak}|}{0.948|V_{\rm peak}|} = 0.5$$

$$\therefore \quad |V_{\rm 20Hz}| = 63.7|V_{\rm 770Hz}|$$

Thus, the 20Hz signal can be 63.7 times as large as the DTMF tones.