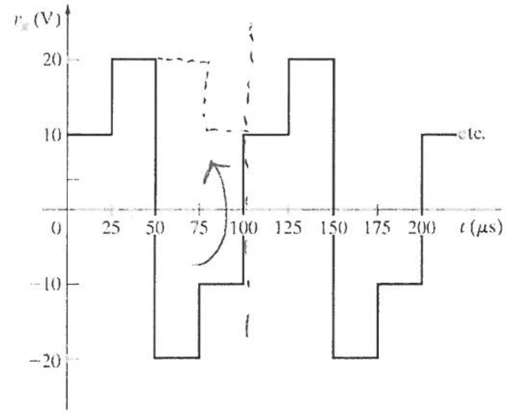
FALL 2013 EENG382 EXAM #1 NAME KEY

Problem #1 (12 pts)

10.15 a) Find the rms value of the periodic voltage shown in Fig. P10.15.

b) If this voltage is applied to the terminals of a 4 Ω resistor, what is the average power dissipated in the resistor?

Figure P10.15



$$v_{RMS}^2 = \frac{1}{2}(10V)^2 + \frac{1}{2}(20V)^2$$

$$v_{RMS}^2 = 50V^2 + 200V^2$$

$$v_{RMS} = \sqrt{250V^2} = 15.81V \leftarrow \text{(\#1a)}$$

$$P_{AVG} = \frac{v_{RMS}^2}{R} = \frac{250V^2}{4\Omega} = 62.5W \leftarrow \text{(\#1b)}$$

Problem #2 (16 pts)

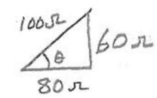
A balanced Y-connected load having an impedance of (80+j60) Ω/φ is connected to a Δ-connected source in which each generator is producing 10kV (rms).

a) What is the apparent power produced by the generator set?

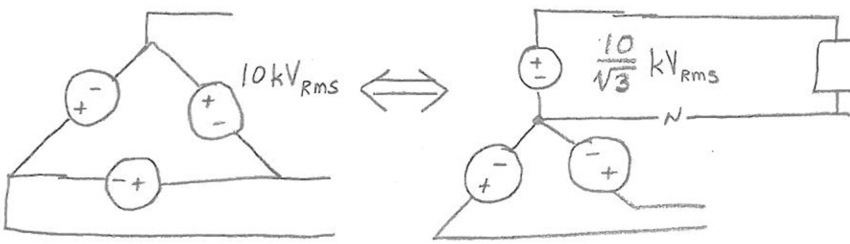
b) What is the real power consumed by the load?

c) What is the complex power produced by the generator set?

b) What is the power factor of the load?



$\cos(\theta) = 0.8$



$(80+j60)\,\Omega = 100\,\Omega\ \angle \underbrace{\operatorname{atan}\left(\frac{60\,\Omega}{80\,\Omega}\right)}_{\theta}$

SINGLE PHASE EQUIVALENT

$\overline{V}_{Rms} = \frac{10}{\sqrt{3}}\,kV \angle \theta$

$\overline{I}_{Rms} = \frac{10\,kV}{\sqrt{3}\cdot 100\,\Omega\angle\theta} = \frac{1}{10\sqrt{3}}\,kA \angle -\theta$

$|\overline{S}|_{PH} = V_{Rms}\cdot I_{Rms} = \left(\frac{10}{\sqrt{3}}\,kV\right)\left(\frac{1}{10\sqrt{3}}\,kA\right) = \frac{1}{3}\,MVA$ (PER PHASE)

$|\overline{S}|_{TOT} = 3\cdot|\overline{S}|_{PH} = 3\cdot\frac{1}{3}\,MVA = 1\,MVA \leftarrow$ (2A)

$pf = \cos(\theta) = 0.8$

$P_{AVG} = S\cdot pf = 1\,MVA\cdot 0.8\,\frac{W}{MVA} = 800\,kW \leftarrow$ (2B)

$\overline{S} = |\overline{S}|\angle\theta = |\overline{S}|\cos\theta + j|\overline{S}|\sin(\theta)$

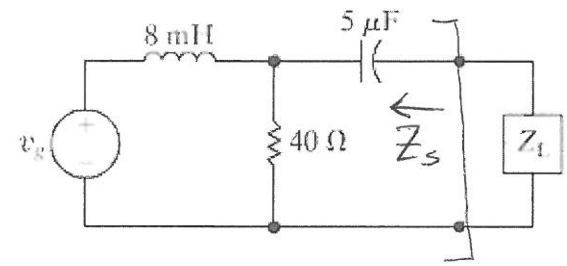
$\overline{S} = 1MVA\cdot 0.8 + j\,1MVA\cdot 0.6 = 800kW + j600kW \leftarrow$ (2C)

$pf = \cos(\theta) = 0.8$ (lagging) $\leftarrow$ (2D)

Problem #3 (12 pts)

10.44 a) Determine the load impedance for the circuit shown in Fig. P10.44 that will result in maximum average power being transferred to the load if $\omega = 5$ krad/s.

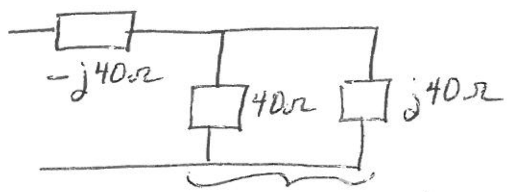
Figure P10.44



$$5\mu F \Rightarrow \frac{1}{j\,5k\,r/s \cdot 5\mu F} = -j40\Omega$$

$$8mH \Rightarrow j\,5k\,r/s \cdot 8mH = j40\Omega$$

$\overline{Z}_s =$



$$\frac{(40\Omega)(j40\Omega)}{(40\Omega + j40\Omega)} = 40\Omega\frac{j(1-j)}{(1+j)(1-j)} = 20\Omega(1+j)$$

$$\overline{Z}_S = 20\Omega(1+j) - j40\Omega = 20\Omega(1+j-j2)$$

$$= 20\Omega(1-j)$$

$$\underline{\underline{\overline{Z}_L = \overline{Z}_s^* = 20\Omega(1+j)}} \leftarrow (\#3)$$

Problem #4 (10 pts)

Find the one-sided Laplace transform of the following function beginning with the definition of the one-sided Laplace transform.

$e^{-at}\sin\omega t$ (damped sine)

$$\mathcal{L}\{e^{-at}\sin(\omega t)\} = \int_{0^-}^{\infty} e^{-at}\sin(\omega t)e^{-st}\,dt$$

$$= \int_{0^-}^{\infty} \frac{e^{j\omega t}-e^{-j\omega t}}{2j}\, e^{-(s+a)t}\,dt$$

$$= \frac{1}{2j}\int_{0^-}^{\infty} e^{-(s+a-j\omega)t} - e^{-(s+a+j\omega)t}\,dt$$

$$= \frac{1}{2j}\left[\frac{-1}{s+a-j\omega}e^{-(s+a-j\omega)t} - \frac{-1}{s+a+j\omega}e^{-(s+a+j\omega)}\right]_{0^-}^{\infty}$$

$$= \frac{1}{2j}\left[\frac{1}{s+a-j\omega} - \frac{1}{s+a+j\omega}\right]$$

$$= \frac{1}{2j}\left[\frac{(s+a+j\omega)-(s+a-j\omega)}{(s+a-j\omega)(s+a+j\omega)}\right]$$

$$= \frac{1}{2j}\left[\frac{\cancel{2(s+a)}\ 2j\omega}{(s+a)^2+\omega^2}\right]$$

$$\underline{\underline{\mathcal{L}\{e^{-at}\sin(\omega t)\} = \frac{\omega}{(s+a)^2+\omega^2}}} \longleftarrow (\#4)$$

Problem #5 (10 pts)

Prove/derive the following operational Laplace transform.

$tf(t)$ $\qquad\qquad$ $-\dfrac{dF(s)}{ds}$

$$F(s) = \int_{0^-}^{\infty} f(t)\, e^{-st}\, dt$$

$$\frac{dF(s)}{ds} = \frac{\partial}{\partial s} \int_{0^-}^{\infty} f(t)\, e^{-st}\, dt$$

$$= \int_{0^-}^{\infty} \frac{\partial}{\partial s}\left(f(t)\, e^{-st} \right) dt$$

$$= \int_{0^-}^{\infty} f(t) \frac{\partial}{\partial s} e^{-st}\, dt$$

$$= \int_{0^-}^{\infty} f(t)\,(-t)\, e^{-st}\, dt$$

$$= -\int_{0^-}^{\infty} t f(t)\, e^{-st}\, dt$$

$$= -\mathcal{L}\{t f(t)\}$$

$$\therefore\ \mathcal{L}\{t f(t)\} = -\frac{dF(s)}{ds} \longleftarrow (\#5)$$